RANKING ORGANIZATIONAL RESILIENCE FACTORS IN ENTERPRISES USING A MODIFIED FUZZY ANALYTICAL HIERARCHY PROCESS

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In the last few decades, the concept of organizational resilience has been developed, and in that manner, it represents an adequate management method for enterprises operating in the conditions of the economic crisis. In this paper, the problem of organizational resilience factors prioritization (ORF) is considered, which represents the first step in the determining of improvement strategies. The ORF ranking is set up as a task of a multi-criterion optimization inclusive of qualitative variables. Management teams at the level of every company assess the relative importance of each pair of business processes and the preference of the ORF within each business process using predefined linguistic expressions. The modeling of linguistic expressions is based on the theory of fuzzy sets. The aggregated values of the considered variables are obtained by applying the fuzzy ordered weighted averaging operator. The vector of a combined ORF priority was determined by using the fuzzy Analytic Hierarchy Process. The proposed model is illustrated by an illustrative example where the used data is obtained from the process of small and medium enterprises (SMEs) operating in central Serbia. It is shown that the developed model is very suitable for making decisions on changing business strategies in order to increase organizational resilience.

Keywords: organizational resilience factors, fuzzy set, fuzzy ordered weighted averaging operator (FOWA), fuzzy Analytic Hierarchy Process (FAHP)

JEL Classification: C61, D80, O21

INTRODUCTION

On the basis of good practice, it can be said that it is almost impossible to identify all the potential factors that may lead to the formation of one or more business risks (Spekman, 2004). Modern business has become very complex, this being caused by the development of new technologies, especially information and communication technologies (ICT). Complex and changeable market conditions are important risk factors that should be managed in the long term with an intention to ensure the sustainable development of an organization (Afgan et al, 2009). Identification, assessment and risk management in almost all organizations are based on the application of the ISO 31000:2008 risk management standard (principles and guidelines) as well as the BS 25999:2006 management and business continuity standard. Recently, the
application of these standards has shown to be insufficient for managing risks successfully because organizations may be faced with very serious risks that may impede management to provide sustainable development.

The subject of the research presented in this paper can be defined as the identification and ranking of organizational resilience factors (ORF). These factors provide a clear market position of an organization, and the ORF rank provides an opportunity for each organization to learn from the experience of other similar organizations.

In this paper, the analyzed organizations belong to the group of the SME process industries. A process industry may be seen as a manufacturing industry using liquid raw materials for the production of finished goods, ensuring a continuous flow of process materials. It should also be noted that final products resulting from the processing industry cannot be broken down into raw materials from which they are made. Bearing in mind the continuity and interconnectedness of the process, it is important to improve the organizational resilience of this type of organizations.

Although there are a number of large multinational organizations in the field of process industries having sufficient resources to sustain the market, a large number of organizations in a process industry fit into the category of SMEs. The characteristics of SMEs can be expressed as follows (Article 2 of Annex Recommendation 2003/361/EC): the number of employees in SMEs is less than 250, the annual income of one SME is no bigger than EUR 50 million, and / or business assets in the balance sheet are less than or equal to EUR 43 million. It is clear that SMEs are recognized as an important economic sector for developing countries and for the developed countries tending to stimulate the entrepreneurial spirit leading to: (a) the creation of new jobs in the process industry, (b) the improvement of product quality (c) the development of a new research whose results should improve processes in the process industry, (d) the usage of advanced technologies (eco-innovation, green technologies etc) and (e) the creation of new markets.

The aim of the research can be defined as prioritizing management initiatives in an exact way that should lead to the improvement of an organization’s performance and its improved sustainability over time. Sustainable development of complex systems is achieved through a constant interaction of business processes and environments. As each organization operates in a highly variable environment, the management is expected to quickly adapt the organization to new business conditions or enable a high level of organizational resilience potential to successfully respond to the changes that have occurred.

This paper defines two basic hypotheses: (1) the level of organizational resilience of process SMEs can be calculated by 11 ORFs, (2) redefining strategies in order to improve the business performance of companies in a changing environment may be based on the obtained rank of ORFs.

Diverse management problems of enterprises such as the problem of organizational resilience management in SMEs discussed in this paper may be described by a number of different variables. The values of the control variables are based on decision-makers’ estimations. Since the business environment is changing quickly and constantly, there is a realistic assumption that it is almost impossible to describe by the precise numbers of variable values. Describing the uncertain and imprecise values by using linguistic statements provided by decision-makers is close to the human way of thinking. In the literature, there are numerous mathematical theories which sufficiently well quantitatively describe linguistic expressions such as probability theory, fuzzy set theory or the theory of rough sets. Zimmermann (2001) considers applying the theory of fuzzy sets to be the most appropriate for the modeling of linguistic statements. In this paper, all the uncertainty and imprecision in the relative importance of business processes and the relative preference of the ORFs within each business process are described by the linguistic variables (Zadeh, 1975) and are modeled by the triangular fuzzy numbers (Klir & Folger, 1988; Zimmeramnn, 2001).

The solution to the considered problem is obtained by using the extended fuzzy Analytic Hierarchy Process (FAHP). The rank of ORFs represents the input for
the process of redefining the business improvement strategies of the discussed type of SMEs.

The paper is organized as follows: the literature review of the reference domain is shown in Section Two. Modelling uncertainty is described in Section Three. In Section Four, the extended fuzzy AHP method for ranking ORFs with respect to all business processes as well as their weight is displayed. In Section Five, an illustrative example illustrating the developed method is provided. In the illustrative example, the data obtained from the process industry SMEs existing in central Serbia is used. The conclusions are presented in Section Six.

LITERATURE REVIEW

Organizational resilience can be analyzed in different research fields, for example in the field of ecological systems (Folke, 2006) or in the domain of socio-economic systems (Adger, 2000). In the field of engineering, organizational resilience is defined as the ability to recognize and feel changes, disruptions and interruptions after which the system adapts and absorbs the same (Hollnagel et al., 2006). The mentioned differences did not allow the creation of a scientific consensus on the constituent elements of organizational resilience nor an appropriate methodology for its assessment.

Various factors (the development of new technologies, globalization etc.) have affected the increasing expectations of customers around the world. Regarding this, a company from the group of SMEs must be innovative and adapt to the emerged challenges (Lee et al., 2012). To achieve this, the SMEs management needs to combine old and new business models as well as enhance their own organizational resilience. This is a very important issue because the SME sector accounts for a significant part of the economic system in the EU states. Thus, for example in 2012, 99.8% of enterprises were from the group of SMEs (Wymenga et al., 2012). This data indicates that SME sector gathers 67.4% of employees in the EU. The improvement of the organizational resilience of SMEs is determined by the market and the company’s properties. Such organizations have a limited access to resources (Vossen, 1998), which makes them open and vulnerable to external entities, so their management have to define an appropriate strategy and provide resources to improve organizational resilience.

Resilience management can be achieved by applying an appropriate business strategy largely affecting the sustainability of SMEs and having a long-term impact on their business performance (Lengnick-Hall et al., 2011). If the effect of human resources within the organization is taken into account, the concept of organizational resilience should be implemented by leaders. Improvements in the field of human resources should result in improving organizational resilience.

In the process industry, organizational resilience is a relatively well-known concept in large and multinational organizations. One of the main challenges in the scope of organizational resilience is achieving the ability to conduct a continuous monitoring system, which includes the monitoring of specific ORFs in order to determine the limits and position the system (Vidal et al., 2009). In that manner, there are a large number of SMEs showing a need for a simple and reliable tool for assessing the capacity for recovery.

Organizational resilience may be treated as a fuzzy problem (Pendall et al., 2010) due to the fact that a large number of events affecting it may be described by uncertain and imprecise data (e.g. sudden disturbances or very slow changes). In order to make the measured values consistent over time, assessments should be made at the level of business processes. The factors contributing to organizational resilience could be estimated (Dinh et al., 2012), which gives a clear picture of the state of the process and their ability to recover if they face a disorder. There are a number of variable sizes which affect both the resilience of critical processes (Carvalho et al., 2008) and the total organizational resilience. Organizational resilience may be represented by appropriate models with certain variables modeled by fuzzy numbers (Chan, 2011). The main objective of this paper is to define a conceptual model for the evaluation of ORFs, which have the greatest impact on the organizational resilience of the considered SMEs.
The evaluation and ranking of ORFs in the presence of uncertainty in the process industry SMEs can be set up as a task of a multi-criteria optimization (MCO). One of the MCO methods, commonly used for solving management problems in a variety of research fields, is the fuzzy Analytic Hierarchy Process (FAHP) (Weeck et al., 1997), which is an extension of conventional AHP (Saaty, 1990). FAHP allows a holistic way in the management modeling problems, and that is the main difference between the FAHP and the other MCO methods.

In the literature, there are numerous approaches to handling the FAHP. One of the most common approaches for the treatment of the FAHP with triangular fuzzy numbers is developed by Chang (1996). The usage of this approach does not require complex mathematical calculations and has the ability to sufficiently support the human way of thinking. This approach can easily be implemented in the conventional AHP (Kwong & Bai, 2003). The weight vector of business processes and the priorities vectors of the ORFs within each business process are obtained by using a method for comparing the fuzzy numbers (Dubois & Prade, 1980; Bass & Kwakeernak, 1977). The priorities of the ORFs with respect to all business processes and their weights are calculated as in the conventional AHP.

In a number of papers dealing with different management problems, solutions are found using the FAHP (Chang, 1996). The paper (Xi & Qin, 2013) treats the problem of an overall assessment of each product’s quality. Priopriteta The priority determination of organizational performance indicators are considered in T. F. Bozbura and A. Beskese (2007). The ranking of the key influence factors of E-business is discussed in F. Kong and H. Liu (2005). In many papers where the solution is sought by using two or more methods of the MCO, the FAHP is used to find the weight of the considered size (Torfi et al., 2010, Tadic et al., 2013).

Respecting the good characteristic of the FAHP, the authors of this study believe that the evaluation and ranking of the ORFs may be well-conducted by the application of the extended FAHP.

**MODELLING OF UNCERTAINTIES**

In this paper, the linguistic expressions are modeled by using the fuzzy sets theory. The fuzzy set is represented by its membership function which the parameters are shaped for, and the location on the universe of discourse. The membership function shape of a fuzzy set can be obtained based on one’s experience, the subjective belief of decision makers, intuition and the contextual knowledge of the concept modelled (Zimmermann, 2001). In the literature, triangular and trapezoidal fuzzy numbers are commonly used for the modeling of different types of uncertainties. Using these fuzzy numbers does not require complex mathematical calculations, and the accuracy of the results is quite appropriate. According to some authors, using fuzzy sets of higher types and levels has not as yet played a significant role in real applications of the fuzzy sets theory (Klir and Yuan, 1995). The domain of fuzzy sets can be defined on different measurement scales: for instance, a common measurement scale (by analogy to the conventional AHP), measurement scales defined in real line into intervals [0-1] and [1-5].

**The basic definitions of the fuzzy sets theory**

The basic definitions of the fuzzy sets theory are presented according to (Dubois & Prade, 1980; Klir & Folger, 1988; Zimmermann, 2001).

**Definition 1.** Uncertainty can be defined as a lack of relevant information on which a decision maker can qualitatively and quantitatively describe a variable (Zimmermann, 2001).

**Definition 2.** A linguistic variable is a variable whose values are expressed in linguistic terms (Zadeh, 1975).

**Definition 3.** Fuzzy set \( \tilde{A} \) is defined as a set of organized pairs: 
\[ \tilde{A} = \left\{ x, \mu_{\tilde{A}}(x) \right\} \mid x \in X, \ 0 \leq \mu_{\tilde{A}}(x) \leq 1 \] where: 
Fuzzy set \( \tilde{A} \) is defined on the universe set \( X \in R \). In general, set \( X \) can be either finite or infinite. \( \mu_{\tilde{A}}(x) \) is a membership function of fuzzy set \( \tilde{A} \). Each fuzzy set is completely and uniquely determined by its membership function.
Definition 4. Fuzzy number \( \tilde{A} \) is a convex normalized fuzzy set of the real line \( R \) such that: if exist \( x_0 \in R \) such that \( \mu_{\tilde{A}}(x_0) = 1 \), \( \mu_{\tilde{A}}(x) \) is piecewise continuous.

Definition 5. Fuzzy number \( \tilde{A} \) on \( R \) is to be a triangular fuzzy number if its membership function is equal to

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{x-a}{m-a} & x \in [a, m] \\
\frac{d-x}{d-m} & x \in [m, d] \\
0 & \text{ostalo}
\end{cases}
\]

Definition 6. A matrix \( \tilde{M} \) is called a fuzzy matrix if at least one element in \( \tilde{M} \) is a fuzzy number (Bortolan and Degani, 1985).

Definition 7. The operations of fuzzy numbers are based on the theorem set by Dubois and Prade (1980).

Let two fuzzy numbers \( \tilde{A} = \{x, \mu_{\tilde{A}}(x) | x \in R \} \) and \( \tilde{B} = \{y, \mu_{\tilde{B}}(y) | y \in R \} \).

The membership functions of these fuzzy numbers are monotous and subjective from zero to one and \( \ast \) is a continuous binary operation. Then \( \tilde{A} \ast \tilde{B} \) is a fuzzy number denoted such as \( \tilde{C} = \tilde{A} \ast \tilde{B} \). Values in the domain of fuzzy number \( \tilde{C} \) can be calculated as \( z = x \ast y \) and

\[
\mu_{\tilde{C}}(z) = \sup_{x \ast y = z} \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))
\]

Consider two triangular fuzzy numbers \( \tilde{A} = (l_1, m_1, u_1) \) and \( \tilde{B} = (l_2, m_2, u_2) \). Their operational laws are as follows:

1. \( (l_1, m_1, u_1) + (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \)
2. \( (l_1, m_1, u_1) - (l_2, m_2, u_2) = (l_1 - u_2, m_1 - m_2, u_1 - l_2) \)
3. \( (l_1, m_1, u_1) \cdot (l_2, m_2, u_2) = (l_1 : u_2, m_1 : m_2, u_1 : l_2) \)
4. \( \lambda \cdot (l_1, m_1, u_1) = (l_1 \cdot \lambda, m_1 \cdot \lambda, u_1 \cdot \lambda) \)
5. \( (l_1, m_1, u_1)^{-1} = \left( \frac{1}{u_1}, \frac{1}{m_1}, \frac{1}{l_1} \right) \)

Definition 8. Arithmetic defuzzification means extracting a single scalar value from a fuzzy set which most appropriately represents the fuzzy set. The moment rule is the most often applied defuzzification technique. It takes as a representative scalar the projection of the central area under the membership function curve to the x-axis.

The modeling of the relative importance of business processes and the relative preference of organizational resilience factors

It can be assumed that all the business processes and the organizational resilience factors (ORF) for the considered business processes are not usually of the same relative importance. Also, they can be considered as unchanged during the considered period of time. The relative importance of the business processes and the relative preference of the ORF under each business process are provided by SMEs’ decision makers. Decision makers’ judgements are based on their knowledge, experience, results of good practice, defined corporate strategies etc.

In this paper, the relative importance of business processes and the relative preference of the ORF are stated by pair-wise comparison matrices (by analogy to AHP). We think that the judgment of each pair of the treated variables best suits human-decision nature instead of a direct access to the assessment. In the conventional AHP, the measurement scale is defined on real sets into interval \([1, 9]\). The use of the discrete scale of the AHP is simple and easy but not sufficient to take into account uncertainty associated with the mapping of one’s perception to a number (Kwong and Bai, 2003). It is realistic to introduce the assumption that decision makers better express their opinions by linguistic expressions.

The elements of these matrices are linguistic expressions modeled by triangular fuzzy numbers

\[
W_{pp'} = (x; l_{pp'}, m_{pp'}, u_{pp'})
\]

\[
W_{pe'} = (x; l_{pe'}, m_{pe'}, u_{pe'})
\]

respectively,

\( p, p' = 1, \ldots, P; p \neq p'; i, i' = 1, \ldots, I; i \neq i'; e = 1, \ldots, E \)

The lower and upper bounds of the used triangular fuzzy numbers are denoted as \( l_{pp'}, l_{pe'}, u_{pp'}, u_{pe'} \) and modal values \( m_{pp'}, m_{pe'} \) which belong to interval \([1, 5]\).
Value 1 denotes that the relative importance of business processes $p$ and the relative preference ORF organizational resilience factor $i$ compared to business process $p'$, as well as ORF $i'$ is equal. Value 5 denotes that business process $p_i$ as well as ORF $i$ is extremely more important than business process $p'$ as well as ORF $i'$, respectively.

If the strong relative importance of business process $p'$ over business process $p$ holds, then a pairwise comparison scale can be represented by fuzzy number $\tilde{W}_{pp} = \left( \tilde{W}_{pe}^{-1} \right)^{-1} = \left( \begin{array}{ccc} 1 - \frac{1}{u_{pp}^e} & \frac{1}{m_{pp}^e} & \frac{1}{l_{pp}^e} \\ \frac{1}{u_{pp'}^e} & 1 - \frac{1}{m_{pp'}^e} & \frac{1}{l_{pp'}^e} \\ \frac{1}{u_{pp'}^e} & \frac{1}{m_{pp'}^e} & 1 - \frac{1}{l_{pp'}^e} \end{array} \right)$.

In similar way, it may be presented:

$$\tilde{W}_{pp'} = \left( \tilde{W}_{pe}^{-1} \right)^{-1} = \left( \begin{array}{ccc} 1 - \frac{1}{u_{pp'}^e} & \frac{1}{m_{pp'}^e} & \frac{1}{l_{pp'}^e} \\ \frac{1}{u_{pp^e}} & 1 - \frac{1}{m_{pp^e}} & \frac{1}{l_{pp^e}} \\ \frac{1}{u_{pp^e}} & \frac{1}{m_{pp^e}} & 1 - \frac{1}{l_{pp^e}} \end{array} \right).$$

If $p = p'; i = i'$ ($i, i' = 1, \ldots, I; p, p' = 1, \ldots, P$), then the relative importance of business processes and the relative preference of ORF $i'$ under business process $p$, is represented by a single point 1, which is a triangular fuzzy number (1,1,1).

In this paper, the management team uses five linguistic expressions modeled by triangular fuzzy numbers given in the following way:

- very low importance /preference - $\tilde{R}_1 = (1,1,2)$
- low importance/preference - $\tilde{R}_2 = (1,2,3)$
- medium importance/preference - $\tilde{R}_3 = (1,3,5)$
- high importance/preference - $\tilde{R}_4 = (3,4,5)$
- very high importance/preference - $\tilde{R}_5 = (4,5,5)$

**METHODOLOGY**

The concept of an extent analysis is presented in (Chang, 1996). By using this concept, the relative importance of business processes, the relative priorities of ORFs under each business process, and the composite priorities ORFs are calculated. The concept of an extent analysis is presented.

**Mathematical problem statement**

In this paper, we consider numerous processing SMEs. Formally, these SMEs are presented by set $E = \{1, \ldots, e, \ldots, E\}$. The index for an SME is denoted as $e$, and $E$ is the total number of the considered SMEs. The management team of each SME consists of: the owner, a production manager, and a financial manager. It can be assumed that the management team at the SME level make decisions by consensus. All the considered SMEs are grouped into $G$ different groups which can formally be presented as $G = \{1, \ldots, g, \ldots, G\}$. The index for an SMEs group is denoted as $g$, $g = 1, \ldots, G$. In this paper, the SMEs are grouped by using the ABC classification method based on Pareto analyses. The SMEs are divided into three groups: A, B, and C, according to the realized annual profit. Typically, the A-class SMEs account for about 5-10% of the number of the considered SMEs, and the management teams of these SMEs receive the highest importance of the assessment. The B-class SMEs account for about 15 percent in terms of the number of the treated SMEs. The management teams of these SMEs have medium importance for the assessment of the production process quality. All the other SMEs belong to Class C, which accounts for about 80 percent of the total number of THE considered SMEs and only about 5 percent of the SMEs’ annual profit value. The management teams of this group of SMEs have low importance for the assessment of the considered problem. The importance of each group of management teams $w_g$, $g = 1, \ldots, G$ is determined based on the results of good practice in the SME processing industry. Also, it can be mentioned that decision makers under the management team are equally important.

Formally, the business processes of the considered SMEs are presented by set $P = \{1, \ldots, p, \ldots, P\}$. The total number of business processes is $P$, and $p$ is an index for a business process. The ORFs are presented by set $I = \{1, \ldots, i, \ldots, I\}$. The index for ORF is denoted as $i$, and $I$ is the total number of the identified ORFs. The fuzzy ratings of the relative importance of the business processes and the relative preference of the ORFs under each business process are given by each member of the management team at the SME.
level. These fuzzy ratings are modelled by triangular fuzzy numbers
\[ \tilde{W}_{pp'} = (x; l_{pp'}, m_{pp'}, u_{pp'}) \]
\[ \tilde{W}_{ii'} = (x; l_{ii'}, m_{ii'}, u_{ii'}) \]
\[ p, p' = 1, \ldots, P; \quad p \neq p'; i, i' = 1, \ldots, I; \quad i \neq i' \]

Since the decision makers belonging to the management team of each SME have the same importance, the relative importance of the business processes and the relative preference of ORFs under each business process are determined by the average value method. These values are modelled by triangular fuzzy numbers according to the fuzzy algebra rules (Klir & Folger, 1988; Zimmermann, 2001). As mentioned, the management teams have unequal importance, so the aggregated values of the relative importance of the business processes, and the aggregated values of the relative preference ORFs, are calculated by using the fuzzy ordered weighted averaging operator (FOWA) explained in (Merigó & Casanovas 2008). These values are denoted as:
\[ \tilde{w}_{pp'} = (x; l_{pp'}, m_{pp'}, u_{pp'}) \]
\[ \tilde{w}_{ii'} = (x; l_{ii'}, m_{ii'}, u_{ii'}) \]
\[ p, p' = 1, \ldots, P; \quad p \neq p'; i, i' = 1, \ldots, I; \quad i \neq i' \]

By using the FAHP (Chang, 1996) weights vector of the identified business processes,
\[ V_p = \left[ w_p \right]_{1 \times P}, \quad p = 1, \ldots, P \]
and the weights vector of the ORFs under each business process, \[ V_{ip} = \left[ w_{ip} \right]_{1 \times I}, \quad i = 1, \ldots, I; \quad p = 1, \ldots, P \]
are given. The value of the combined priority index \[ r_i \]
\[ i = 1, \ldots, I \] is associated to each ORF and based on that, the ORH ranking is done. The ORF associated with the largest or the smallest value is ranked in the first place, or the last. The first-place rank located ORF has the biggest impact on the organizational resilience of the considered SMEs. In other words, on the basis of the ORF’s obtained rank, the priority of management initiatives that should lead to an organizational resilience enhancement in SMEs is determined.

The Algorithm of the extended FAHP

The problem of the evaluation and ranking of the ORFs in processing SMEs by a modified FAHP is shown.

Step 1. The considered problem is decomposed into several less complex management problems. The fuzzy pair-wise comparison matrix of the relative importance of the business processes,
\[ \tilde{W}_p^e = \left[ \tilde{W}_{pp'} \right]_{P \times P}, \quad p = 1, \ldots, P; \quad e = 1, \ldots, E \]
and the fuzzy matrix of the priorities of the ORFs with respect to each treated business process,
\[ \tilde{W}_i^{pe} = \left[ \tilde{W}_{ii'} \right]_{I \times I}, \quad i = 1, \ldots, I; \quad p = 1, \ldots, P; \quad e = 1, \ldots, E \]
are constructed.

Step 2. Calculate the aggregated values of the elements of the constructed pair-wise comparison matrices for each group of SMEs:
\[ \tilde{W}_p^e = \frac{1}{E_g} \sum_{e=1}^{E_g} \tilde{W}_{pp'}^e, \quad p = 1, \ldots, P; \quad e = 1, \ldots, E \]
\[ \tilde{W}_i^{pe} = \frac{1}{E_g} \sum_{e=1}^{E_g} \tilde{W}_{ii'}^{pe}, \quad i = 1, \ldots, I; \quad p = 1, \ldots, P; \quad e = 1, \ldots, E \]

Step 3. Calculate the aggregated relative importance of the business processes and the ORFs under each business process by using the FOWA:
\[ \tilde{w}_{pp'} = \text{FOWA} \left( \tilde{W}_p^e \right) = \sum_{g=1}^{G} w_g \cdot \tilde{W}_p^e, \]
\[ \tilde{w}_{ii'} = \text{FOWA} \left( \tilde{W}_i^{pe} \right) = \sum_{g=1}^{G} w_g \cdot \tilde{W}_i^{pe}, \]
\[ p = 1, \ldots, P; \ i = 1, \ldots, I; \ e = 1, \ldots, E_g \]
where \[ E_g \]
\[ g = 1, \ldots, G \] is the total number of the management teams of the g-group SMEs.

Step 4. Determine the pair-wise comparison matrix of the relative importance of the business processes,
\[ \tilde{W}_p = \left[ \tilde{W}_{pp'} \right]_{P \times P}, \quad p = 1, \ldots, P \]
and the pair-wise comparison matrix of the relative preference of the ORFs under each business process,
\[ \tilde{W}_i = \left[ \tilde{W}_{ii'} \right]_{I \times I}, \quad p = 1, \ldots, P; \ i = 1, \ldots, I \]

D. Tadic, A. Aleksic  
Ranking organizational resilience factors in enterprises using a modified fuzzy analytical hierarchy process
The representative scalars of the triangular fuzzy numbers are given by using the moment method (Klir and Folger, 1988).

Step 5. The consistency of the fuzzy pair-wise comparison matrices \( W_p = [w_{pp'}]_{p,p} \) and \( W_i = [w_{ii'}]_{i,i} \) should be checked. A necessary condition for consistency is that these matrices be reciprocal. A sufficient condition for consistency is that the principal eigenvalue of each matrix, \( \lambda_{\text{max}} \) be equal to dimension of matrix (Saaty, 1990). The eigenvector method also yields a natural measure for inconsistency. The consistency index (C.I.) of matrices \( W_p \) and \( W_i \), \( p = 1,...,P; i = 1,...,I \) can be calculated as:

\[
\text{C.I.} = \frac{C.R.}{R.I.} = \frac{\lambda_{\text{max}} - I}{I - 1}
\]

where:

\[
C.R. = \frac{\lambda_{\text{max}} - P}{P - 1}, \quad C.R. = \frac{\lambda_{\text{max}} - I}{I - 1}
\]

and an eigenvectors of matrices \( W_p \) and \( W_i \), \( p = 1,...,P; i = 1,...,I \) are denoted as \( \lambda_{\text{max}} \) and \( \lambda_{\text{max}} \), respectively.

The eigenvalues of vectors can be calculated:

\[
W_{i,p} \cdot \lambda_{\text{max}} = \lambda_{\text{max}} \cdot (W_i)p', \quad (W_p)_{i,p} \cdot \lambda_{\text{max}} = \lambda_{\text{max}} .
\]

The elements of matrices \( W_{i,p} \), \( (W_i)_{i,p} \), \( i = 1,...,I; p = 1,...,P \) are calculated by using a linear normalization procedure for benefit-type variables (Hwang & Yoon, 1981).

A random index (R.I.) is defined for each size of matrix as presented in Table 1 (Vargas, 1982).

The number 10% is the accepted upper limit for C.I. If C.I. > 0.1, it is recommended that the management team should revise some assessments since they are highly inconsistent. The method for choosing which assessments should be considered for revision in order to reduce inconsistency is proposed in (Harkar, 1987).

Step 6. The vector weights of the identified business processes, \( V_p = [w_{pp'}]_{1,P} \), \( p = 1,...,P \) and the vector preference of the ORFs under each business process, \( V_i = [w_{ii'}]_{1,I} \), \( i = 1,...,I; p = 1,...,P \) are calculated by using the concept of an extent analysis (Chang, 1996).

Let \( X = \{x_1,...,x_n\} \) be an object set, and \( Y = \{y_1,...,y_n\} \) be a goal set. According to the concept of the extent analysis (Chang, 1996), each object is taken and the extent analysis for each goal is performed, respectively. Therefore, the P extent analysis values for each object are marked by the following signs:

\[
N_i^j, \quad j = 1,...,P, \quad i = 1,...,I
\]

The value of the fuzzy synthetic extent with respect to the \( i \)-th object is defined as:

\[
\tilde{S}_i = \sum_{j=1}^{P} N_i^j \left[ \sum_{i=1}^{P} \sum_{j=1}^{P} N_i^j \right]^{-1}
\]

where:

\[
\sum_{j=1}^{P} N_i^j = \left( \sum_{p=1}^{P} l_{pp'} \cdot \sum_{p=1}^{P} u_{pp'} \cdot \sum_{p=1}^{P} m_{pp'} \right)
\]

\[
\sum_{i=1}^{P} \sum_{j=1}^{P} N_i^j = \left( \sum_{p=1}^{P} \sum_{p=1}^{P} l_{pp'} \cdot \sum_{p=1}^{P} \sum_{p=1}^{P} m_{pp'} \cdot \sum_{p=1}^{P} \sum_{p=1}^{P} u_{pp'} \right)
\]

### Table 1 Random index (RI)

<table>
<thead>
<tr>
<th>Size of matrix</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>R.I.</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
<td>1.51</td>
<td>1.48</td>
<td>1.56</td>
<td>1.57</td>
</tr>
</tbody>
</table>

Source: Vargas, 1982
The weights vector of the identified business processes is represented as:

\[
\left(\text{Bel}(\bar{S}_1), \ldots, \text{Bel}(\bar{S}_i), \ldots, \text{Bel}(\bar{S}_p)\right)
\]

Where \(\text{Bel}(\bar{S}_i)\) is the measure of the belief according to which triangular fuzzy number \(\bar{S}_i\) is bigger than all other triangular fuzzy numbers \(\bar{S}_{i'}\), \(i, i' = 1, \ldots, P; i \neq i'\). This value is obtained by applying the method for fuzzy numbers comparison (Dubois & Prade, 1980; Bass & Kwakernaak, 1977) (see Appendix).

After normalizing, the normalized weights vector of the identified business processes \(V_p\) is obtained:

\[
V_p = \left(\frac{w_1}{w_p}, \ldots, \frac{w_p}{w_p}, \ldots, \frac{w_p}{w_p}\right)
\]

\(w_p\) is a non-fuzzy number, which gives the priority to the weights of one business process over the other.

In a similar way, the normalized preference vector of the ORFs under each business process is obtained.

Step 7. Calculate to composite priorities for each ORF, \(r_i, i = 1, \ldots, I\):

\[
r_i = \sum_{p=1}^{P} w_p \cdot \frac{r_i}{w_p}, i = 1, \ldots, I; p = 1, \ldots, P.
\]

Step 8. Rank all \(r_i, i = 1, \ldots, I\) in the decreasing order.

**ILLUSTRATIVE EXAMPLE**

The improvement of organizational resilience is a very important issue for the management teams of the process industry SMEs in developing countries. The proposed method is illustrated with the real data obtained in the 53 process industry SMEs operating in central Serbia. The considered SMEs are certified according to the requirements of the ISO 9001, necessary for defining the reference model of organization. As the ranking of organizational resilience factors is done at the level of business processes, it is important to define the business processes in the observed group of SMEs (Oakland, 2004). The processes in this type of organization are: management (p=1), production (p=2), marketing and sales (p=3), purchase (p=4), design and development (p=5), and support processes (p=6).

The ORFs of SMEs in processing industry are presented in (Aleksić et al., 2013). These factors are:

- planning strategies - the factor whose influence is dominant in the process of management and strategy activities;
- the capability and capacity of internal resources - the factor scoped to internal processes;
- the internal situation monitoring and reporting - the factor integrated into internal processes;
- human factors - the factor whose influence is dominant in the scope of human resources;
- quality - the factor integrated into all business processes;
- the external situation monitoring and reporting - the factor integrated into the external processes;
- the capability and capacity of external resources - the factor scoped to external processes;
- the design factor - the factor integrated into manufacturing operations;
- a detection potential - the factor whose influence is dominant in processes sensitive to deviations from desired goals;
- an emergency response - the factor whose influence is dominant in the period of a crisis;
- a safety management system - the factor integrated into the activities related to the safety of employees and company property.

All the considered SMEs are sorted out according to the annual income of the monotonous descending order. According to the result of the classification, 5 SMEs belong to Group A, 19 SMEs belong to Group B whereas Group C consists of 29 SMEs. The weights of the given groups of SMEs are determined with respect to the knowledge and experience of experts analyzing the strength of the enterprises influence on the realization of the country development strategy.

In this case, the weights of the groups are \(w_A = 0.45\), \(w_B = 0.35\), \(w_C = 0.2\).

The element values of the constructed fuzzy pairwise comparison matrices are given by applying the proposed Algorithm (Step 1 to Step 3). The developed procedure is illustrated by an example of the real-life
of the management process \((p=1)\) and the marketing and sales process \((p=2)\).

The relative importance of the two considered business processes is assessed by the management teams of the SMEs belonging to the following groups: medium importance, high importance \(x3\) and very high importance. The fuzzy rating of the Group B management teams are: low importance \(x2\), medium importance \(x4\), high importance \(x10\) and very high importance \(x4\). The fuzzy ratings of the Group C management teams are: medium importance \(x11\), high importance \(x11\) and very high importance \(x5\).

The relative importance of the considered business processes for each group is as follows:

\[
\tilde{W}_{12}^A = \frac{1}{5} (\tilde{R}_3 + 3 \cdot \tilde{R}_4 + \tilde{R}_5) = (2.8, 4.5),
\]

\[
\tilde{W}_{12}^B = \frac{1}{19} (\tilde{R}_2 + 4 \cdot \tilde{R}_3 + 10 \cdot \tilde{R}_4 + 4 \cdot \tilde{R}_5) = (2.68, 3.89, 4.89)
\]

\[
\tilde{W}_{12}^C = \frac{1}{29} (11 \cdot \tilde{R}_3 + 11 \cdot \tilde{R}_4 + 5 \cdot \tilde{R}_5) = (2.21, 3.52, 5)
\]

The relative importance of the management process \((p=1)\) and the relative importance of the marketing and sales process \((p=2)\) for all the 53 SMEs in processing industry can be calculated as:

\[
\tilde{w}_{12} = 0.45 \cdot \tilde{W}_{12}^A + 0.35 \cdot \tilde{W}_{12}^B + 0.2 \cdot \tilde{W}_{12}^C = (2.28, 3.87, 5)
\]

In a similar way, the element values of the constructed fuzzy pair-wise comparison matrices are calculated.

The developed procedure of the proposed Algorithm (Step 4 to Step 6) is illustrated by example for checking the consistency of the pair-wise comparison matrix of the relative importance of the considered business processes.

By using the proposed Algorithm (Step 4), the crisp pair-wise comparison matrix of the relative importance of the business processes is given:

\[
W_p = \begin{bmatrix}
1 & 3.87 & 4.39 & 3.82 & 1.97 & 2.28 \\
0.53 & 1 & 1.35 & 0.98 & 0.44 & 0.99 \\
0.23 & 0.74 & 1 & 0.58 & 0.39 & 0.43 \\
0.26 & 1.02 & 1.72 & 1 & 0.64 & 1.89 \\
0.51 & 2.27 & 2.56 & 1.57 & 1 & 3.97 \\
0.44 & 1.01 & 2.33 & 0.53 & 0.25 & 1
\end{bmatrix}
\]

Let us check to consistency of the above fuzzy matrix (Step 5 of the proposed Algorithm).

Applying the proposed algorithm for checking matrix consistency started in (Harker, 1988), the weights vector in the first iteration is calculated:

\[
w_p = \begin{bmatrix}
0.337 & 0.391 & 0.329 & 0.450 & 0.420 & 0.215 \\
0.178 & 0.101 & 0.101 & 0.116 & 0.094 & 0.094 \\
0.077 & 0.075 & 0.075 & 0.068 & 0.083 & 0.041 \\
0.088 & 0.103 & 0.129 & 0.118 & 0.136 & 0.179 \\
0.172 & 0.229 & 0.192 & 0.185 & 0.213 & 0.376 \\
0.148 & 0.102 & 0.175 & 0.062 & 0.053 & 0.095
\end{bmatrix}
\]

\[
(w_p)^T = \begin{bmatrix}
0.357 \\
0.114 \\
0.070 \\
0.125 \\
0.259 \\
0.106
\end{bmatrix}
\]

Applying the proposed algorithm for checking matrix consistency started in (Harker, 1988), the weights vector in the first iteration is calculated:

\[
w_p = \frac{A \cdot e}{e^T \cdot A \cdot e} = \begin{bmatrix}
0.347 & 0.106 & 0.067 & 0.131 & 0.238 & 0.111
\end{bmatrix}^T
\]

It is obvious that consistency has not been achieved in the first operation. Continuing this process, we have thus the process has converged in five iterations:

\[
w_p = \begin{bmatrix}
0.361 & 0.098 & 0.071 & 0.129 & 0.236 & 0.105
\end{bmatrix}^T
\]

Checking the consistency of the pair-wise matrices of the preference of the ORFs under each business process is performed by using the procedure presented in Step 4 of the proposed Algorithm.

The obtained results accounted for by applying the proposed Algorithm (Step 1 to Step 6) are presented in Table 2.
The composite priorities are calculated by using the proposed Algorithm (Step 7 to Step 8).

Based on the calculated values of the combined index of priorities, it can be concluded that the ORFs with the biggest impact on the organizational resilience of the process industry SMEs are the planning strategies ($i = 1$) and the quality ($i = 5$). The obtained result shows that, in the first place, it is necessary to deploy management initiatives that should lead to an increase in the two named ORFs, e.g.: a continuous process of improvement, an improvement of communication between all employees in the company, or an improvement of communication between the management teams and stakeholders. The effectiveness of management measures is empowered if the enterprise uses information and communication technology (ICT). On the other hand, the ORF, designated as a potential detection ($i = 9$), has the smallest impact on organizational resilience, which is realistic due to the fact that the considered SME quality system does not contain a large number of procedures with respect to the size of the enterprise.

**CONCLUSION**

Changes occurring in an uncertain business environment require that new management concepts whose task is to increase the effectiveness of the business and sustainability goals of an organization over time should be developed and implemented.

### Table 2 The business process weights and priorities of ORFs under each business process

<table>
<thead>
<tr>
<th>i</th>
<th>p=1</th>
<th>p=2</th>
<th>p=3</th>
<th>p=4</th>
<th>p=5</th>
<th>p=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=1</td>
<td>0.01</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>i=2</td>
<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
<td>0.03</td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>i=3</td>
<td>0.07</td>
<td>0.08</td>
<td>0.06</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>i=4</td>
<td>0.08</td>
<td>0.08</td>
<td>0.07</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>i=5</td>
<td>0.17</td>
<td>0.13</td>
<td>0.15</td>
<td>0.14</td>
<td>0.12</td>
<td>0.16</td>
</tr>
<tr>
<td>i=6</td>
<td>0.04</td>
<td>0.09</td>
<td>0.15</td>
<td>0.05</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>i=7</td>
<td>0.12</td>
<td>0.11</td>
<td>0.09</td>
<td>0.10</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>i=8</td>
<td>0.08</td>
<td>0.07</td>
<td>0.05</td>
<td>0.09</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>i=9</td>
<td>0.19</td>
<td>0.19</td>
<td>0.23</td>
<td>0.23</td>
<td>0.14</td>
<td>0.12</td>
</tr>
<tr>
<td>i=10</td>
<td>0.05</td>
<td>0.05</td>
<td>0.02</td>
<td>0.04</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>i=11</td>
<td>0.15</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>The business process weights</td>
<td>0.36</td>
<td>0.09</td>
<td>0.07</td>
<td>0.12</td>
<td>0.23</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Source:** Authors

### Table 3 The values of the composite priorities and rank of the ORFs

<table>
<thead>
<tr>
<th>i</th>
<th>$\rho_i$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>i=1</td>
<td>0.1480</td>
<td>1</td>
</tr>
<tr>
<td>i=2</td>
<td>0.0533</td>
<td>10</td>
</tr>
<tr>
<td>i=3</td>
<td>0.070</td>
<td>7</td>
</tr>
<tr>
<td>i=4</td>
<td>0.078</td>
<td>6</td>
</tr>
<tr>
<td>i=5</td>
<td>0.146</td>
<td>2</td>
</tr>
<tr>
<td>i=6</td>
<td>0.058</td>
<td>8</td>
</tr>
<tr>
<td>i=7</td>
<td>0.114</td>
<td>4</td>
</tr>
<tr>
<td>i=8</td>
<td>0.089</td>
<td>5</td>
</tr>
<tr>
<td>i=9</td>
<td>0.0348</td>
<td>11</td>
</tr>
<tr>
<td>i=10</td>
<td>0.056</td>
<td>9</td>
</tr>
<tr>
<td>i=11</td>
<td>0.127</td>
<td>3</td>
</tr>
</tbody>
</table>

**Source:** Authors
The first hypothesis is confirmed according to the requirements of the ASIS SPC.1-2009 standard compatible with the series of the ISO 9001, ISO 14001 and ISO 27001 standards. The number of the ORFs, measured in this paper within the scope of organizational resilience, is defined in terms of the discussed organization types, according to Aleksić et al. (2013). The second hypothesis is theoretically accurate but it should be confirmed in a future practice since the measures for redefining the strategy should bring results in the following period.

Modelling the relative importance of the business processes and the relative preference of the ORFs is based on the usage of the fuzzy sets. The fuzzy approach is easy to understand, flexible and tolerant to inaccurate data.

The evaluation and ranking of the ORFs represent one of the most important management problems of organizational resilience in all organizations in a changing and uncertain environment. According to the authors, the proposed fuzzy AHP is an appropriate method for determining the rank of the ORFs in an exact way. The priority obtained is less burdened by the subjective attitudes of decision makers, stakeholders and so forth, so it can be considered that an improvement of the strategy effectiveness will be higher, which is one of the main objectives of the management team of any enterprise engaged in various economic activities.

The contributions of this paper can be expressed as follows: (1) identifying the business processes and the ORFs for process industry SMEs, (2) the treatment of uncertainty in the relative importance of relations between business processes and the relative priorities of ORFs was performed by using the fuzzy sets theory, (3) the aggregation of the management teams’ assessment into the group consensus is obtained by applying an operator of the fuzzy-weighted mean value, (4) the ranking of the selected ORFs corresponds to the values of the combined priorities index, and (5) the rank of ORFs allows enterprises to learn over time, which increases the effectiveness of their business processes and continuous development.

Apart from the presented advantages, the proposed model has certain limitations. It can be extended in terms of the better structuring of business processes, increasing the number of ORFs depending on the size of the enterprise and/or the types of the economic activity realized within the enterprise. All of these extensions can easily and quickly be incorporated into the proposed model and do not increase the complexity of mathematical computation.

The proposed model is tested on the group of small and medium enterprises in the process industries operating in central Serbia.

A continuous improvement of business processes (which is one of the basic requirements of ISO 9000:2008) is achieved by developing and applying appropriate strategies for improving each group of the identified indicators measuring the effectiveness of business processes. The results of good practice show that the application of management initiatives, based on the priority indicators, enables an improvement of a business process realized in a shorter period of time and at a significantly lower cost. Priority indicators, for example the ORFs, can be determined in an exact manner, for example by using the method proposed in this paper.

Future research will be focused on the development and/or modification of the approaches that can be found in the reference literature for the processing of fuzzy matrix uncertainties existing in the considered problem. By applying new approaches, the ranking of ORFs is obtained, based on which management teams can make a better analysis of their priorities. The identification of ORFs and the determination of their priorities in public enterprises and service-oriented enterprises are also within the scope of a further research. In the authors’ opinion, the development of the software based on the model may provide a fast and efficient analysis of ORFs priorities in different enterprises.

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REFERENCES


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**Aleksandar Aleksić** is a research associate at the Center for Quality at the Faculty of Engineering, University of Kragujevac, Kragujevac where he received his PhD degree. Since 2008, he has been involved in the process of teaching the subjects: Fundamentals of Entrepreneurial Management and Economics, Engineering Economy, and Management Communications. He has authored or co-authored 1 scientific monograph, 5 papers published in the SCI list journals and more than 30 papers published at national and international conferences.
APPENDIX: COMPARISON OF FUZZY NUMBERS

In this Appendix, a simple method of comparing fuzzy numbers and determining a degree of belief that one fuzzy number is greater than or equal to one is given (Bass & Kwakernaak, 1977; Dubois & Prade, 1980).

Let \( \tilde{A} \) and \( \tilde{B} \) be two fuzzy numbers with their supports defined on \( R \):
\[
\tilde{A} = (x; l_1, m_1, u_1) \quad \text{and} \quad \tilde{B} = (y; l_2, m_2, u_2)
\]
Where \( l_1, l_2, u_1, u_2 \) are lower and upper bounds and \( m_1, m_2 \) are modal values of \( \tilde{A} \) and \( \tilde{B} \), respectively. Let \( m_2 < m_1 \) and \( l_2 < l_1 \) and \( u_2 < u_1 \).

The degree of belief that \( \tilde{B} \) is greater than or equal to \( \tilde{A} \) is denoted by \( Bel(\tilde{B} \geq \tilde{A}) \), which is given using the operation max and min (Dubois & Prade, 1980):
\[
Bel(\tilde{B} \geq \tilde{A}) = \max_{x,y} \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y))
\]
(D.1)

The expression can be defined as:

1. \( Bel(\tilde{A} \geq \tilde{B}) = 1 \), because \( \mu_{\tilde{A}}(m_1) = 1 \) and \( \mu_{\tilde{B}}(m_2) = 1 \) and \( m_1 > m_2 \) (D.2)

2. At the same time, \( Bel(\tilde{B} \geq \tilde{A}) \) is equal to the ordinate of point D, which belongs to both \( \tilde{A} \) and \( \tilde{B} \), i.e. it is the supremum of intersection:
\[
Bel(\tilde{B} \geq \tilde{A}) = \text{the ordinate of point D.} \quad \text{(D.3)}
\]

When \( \tilde{A} \) and \( \tilde{B} \) are triangular fuzzy numbers, the ordinate of D is given by the equation:
\[
Bel(\tilde{B} \geq \tilde{A}) = \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}
\]
(D.4)

For the full understanding of the risk analysis presented in this paper, it is important that the degree of belief that fuzzy number \( \tilde{A} \) is bigger than/equal to K fuzzy numbers \( \tilde{B}_1, ..., \tilde{B}_k, ..., \tilde{B}_K \) (Bass & Kwakernaak, 1977) should be determined:
\[
Bel(\tilde{A} \geq \tilde{B}_1, ..., \tilde{B}_k, ..., \tilde{B}_K) = \\
\sup_{t \in t_k} \min_{t \in t_k} \left( \mu_{\tilde{A}}(t), \mu_{\tilde{B}_1}(t_1), ..., \mu_{\tilde{B}_k}(t_k) \right) \\
\sup_{t \in t_k} \min_{t \in t_k} \left( \mu_{\tilde{A}}(t), \mu_{\tilde{B}_1}(t_1), ..., \mu_{\tilde{B}_k}(t_k) \right) \\
\sup_{t \in t_k} \min_{t \in t_k} \left( \mu_{\tilde{A}}(t), \mu_{\tilde{B}_1}(t_1), ..., \mu_{\tilde{B}_k}(t_k) \right) \\
Bel(\tilde{A} \geq \tilde{B}_1, ..., \tilde{A} \geq \tilde{B}_k, ..., \tilde{A} \geq \tilde{B}_K) = \\
\min_{k=1,...,K} Bel(\tilde{A} \geq \tilde{B}_k)
\]
(D.5)