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# ECONOMIC FORECASTING WITH TANGIBLE AND INTANGIBLE CRITERIA: THE ANALYTIC HIERARCHY PROCESS OF MEASUREMENT AND ITS VALIDATION

***Abstract:*** *This paper provides a summary of a mathematical theory about the use of expert judgments in paired comparisons and how to derive priorities from them particularly when intangible factors are involved. An example to validate the process when applied to tangibles is given along with a simple decision example to determine which city to choose to live that involves several intangible criteria. The paper then deals with three kinds of applications of the process in economics. One is about currency exchange, the second about input-output analysis and the third about forecasting turnaround dates of the US economy in 1992, in 2001 and finally in 2008-2009.*

***Keywords:*** *Intangibles, analytic hierarchy process (AHP), dollars versus the yen, input-output, forecasting*

# EKONOMSKO PREDVIĐANJE PRIMENOM MERLJIVIH I NEMERLJIVIH KRITERIJUMA: ANALITIČKI HIJERARHIJSKI PROCES MERENJA I NJEGOVO VREDNOVANJE

***Abstract:*** *U ovom radu dat je pregled jedne matematičke teorije o stručnoj proceni parnih poređenja i kako na osnovu toga odrediti prioritete, a naročito kada su uključeni nemerljivi faktori. Naveden je jedan primer vrednovanja ovog procesa kada su primenjeni merljivi faktori kao i jednostavan primer odlučivanja o izboru mesta za život kada je uključeno nekoliko nemerljivih kriterijuma. U radu se zatim navode tri vrste primene ovog procesa u ekonomiji. Jedan je u vezi sa deviznom berzom, drugi sa input-output*

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*analizom, a treći u vezi sa predviđanjem preokreta u američkoj ekonomiji 1992, 2001. i na kraju – 2008-2009. godine.*

***Ključne reči:*** *nemerljivi faktori, analitički hijerarhijski proces (AHP), dolar nasuprot jenu, input-output, predviđanje*

**JEL Classification:** C02

## 1. Introduction

According to the philosopher Emanuel Kant in life we have to deal with the answers of exactly three questions: What do I know? What shall I do? What can I expect? The first is a question about the past, the second about the present and the third about the future.There are many intangibles that influence what happens in the future and the role of judgment is essential to surmise their effect.

It is certain that there are numerous factors that influence our lives for which we have no measurements and therefore these intangible factors are not included in scientific theories. We are as strong and knowledgeable as the understanding we get from our measurements. It seems that measurements that leave out many important factors cannot give as much information and knowledge to us as we need. Another important fact is that measurements in the physical world involve arbitrary units using which measurements are made uniformly whether they are small or large. As a result they need experience and understanding to interpret what they mean. One must have considerable experience with any scale of measurement we use to interpret what a number on that scale means. In science we first create measurements then use judgment to interpret them. In addition, tangible factors are mostly concrete entities and last for some duration of time. Intangibles can be influences that happen rapidly and then pass. Human behavior tends to fall in this category. Its strength and effects cannot be held down so we can perform measurements on them. Thus judgment is fundamental for making and intangible measurements.

The Analytic Hierarchy Process (AHP) is a theory of measurement that does the opposite of what is done in science, it uses judgment first to make comparisons of elements with respect to some common property they have and then derives scales of relative priority measurement from these judgments. These priorities no longer need judgment to interpret them. In doing that the AHP can be used to derive measurements for anything and the meaning of the priorities depends on the initial judgments. The question is how does it work and how valid and reliable is it?

Examples of intangibles that impact economics are political and social factors. One purpose may be to quantify them and use the priorties along with economic measurements to get an overall outcome. Complexity is described by many factors that are interdependent and to understand these result of interdependence we need to structure the problem. Strucutrs are made by infomred people through brainstorming and the use of morphorogical analysis that are fundamental components of creative thinking.. We need our collective understanding and experience to create such elaborate structures. We then qunatify the influences of the factors in these structures. It is not so simple as we do in science to write a formula that describes the relations among these factors nor to determine the outcome of their influence. In the end we are the judges of what meaning we seek and how to go about obtaining that meaning. To repeat what we said before, even in science, and contrary to what we have been taught, all measurements need to be interpreted by an expert in the field as to their significance because the units of measurement in yards and meters and in pounds and kilograms are arbitrary and need familiarity to appreciate their quantitative value. In the end, the world is subjective and truth depends on our value system and on what we are after.

The mathematics we use to understand the world involves structures and their connections. These structures come in the form of hierarchies and networks with dependence and feedback. One needs to include all the factors one uses to judge the outcome and derive that outcome based on the factors used .It involves prioritization based on expert judgments and on data that is interpreted by these experts as to their significance. The outcome of an analysis is also in the form of priorties that desigante the importance of various outcomes and the stability of those priorties to small changes in the influences that bring them about. The structures of the AHP must include all the factors used to determine the best outcome. In the AHP, the outcome is a result of the factors included and the judgments used. When it is discovered later that the outcome does not meet certain requirements due to the absence of factors or judgments, those factors must be added and judgments revised to determine the best outcome.

The examples given below deal with economic forecasting all made by experienced and knowledgeable economists. The first is an example of input-output analysis made in the 1970's. The second was to determine the value of the dollar versus the Japanese yen in the late 1980s. The other three examples have to do with the date of recovery of the US economy in 1992, 2001, and 2008.

## 2. The Analytic Hierarchy Process [1]

### 1) Deriving a Scale of Priorities from Pairwise Comparisons

Suppose we wish to derive a scale of relative importance according to size (volume) of three apples A, B, C shown in Figure 1. Assume that their volumes are known respectively as $S_1, S_2$ and $S_3$. For each position in the matrix the volume of the apple at the left is compared with that of the apple at the top and the ratio is entered. A matrix of judgments $A = (a_{ij})$ is constructed with respect to a particular property the elements have in common. It is reciprocal; that is, $a_{ji} = 1/a_{ij}$, and $a_{ii} = 1$. For the matrix in Figure 1, it is necessary to make only three judgments with the remainder being automatically determined. There are $n(n-1)/2$ judgments required for a matrix of order $n$. Sometimes one (particularly an expert who knows well what the judgments should be) may wish to make a minimum set of judgments and construct a consistent matrix defined as one whose entries satisfy $a_{ij}a_{jk} = a_{ik}$, $i, j, k = 1, \ldots n$. To do this one can enter $n-1$ judgments in a row or in a column, or in a spanning set with at least one judgment in every row and column, and construct the rest of the entries in the matrix using the consistency condition. Redundancy in the number of judgments generally improves the validity of the final answer because the judgments of the few elements one chooses to compare may be more biased.

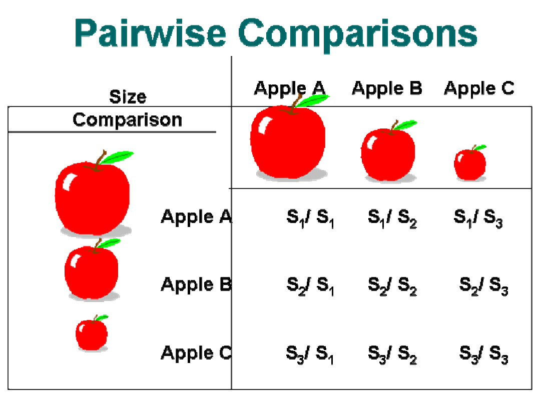



Figure 1. Reciprocal Structure of Pairwise Comparison Matrix for Apples

Assume that we know the volumes of the apples so that the values we enter in Figure 2 are consistent. Apple A is twice as big in volume as apple B, and apple B is three times as big as apple C, so we enter a 2 in the (1,2) position, and so on. Ones are entered on the diagonal by default as every entity equals itself on any criterion. Note that in the (2, 3) position we can enter the value 3 because we know the judgments are consistent as they are based on actual measurements. We can deduce the value this way: from the first row A = 2B and A = 6C, and thus B = 3C.

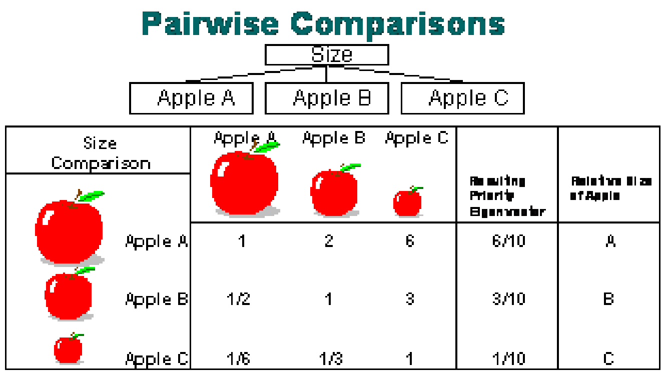



Figure 2. PairwiseComparison Matrix for Apples using Judgments

If we did not have actual measurements, we could not be certain that the judgments in the first row were accurate, and we would not mind estimating the value in the (2, 3) position directly by comparing apple B with apple C. We are then very likely to be inconsistent. How inconsistent can we be before we think it is intolerable? Later we give an actual measure of inconsistency and argue that a consistency of about 10% is considered acceptable. .

We obtain from the consistent pairwise comparison matrix above a vector of priorities showing the relative sizes of the apples. Note that we do not have to go to all this trouble to derive the relative volumes of the apples. We could simply have normalized the actual measurements. The reason we did so is to lay the foundation for what to do when we have no measures for the property in question. When judgments are consistent as they are here, this vector of priorities can be obtained in two ways: dividing the elements in any column by the sum of its entries (normalizing it), or by summing the entries in each row to obtain the overall dominance in size of that alternative relative to the others and normalizing the resulting column of values. Incidentally, calculating dominance plays an important role in computing the priorities when judgments are inconsistent for then an alternative may dominate another by different magnitudes by transiting to it through intermediate alternatives. Thus the story is very different if the judgments are inconsistent, and we need to allow inconsistent judgments for good reasons. In sports, team A beats team B, team B beats team C, but team C beats team A. How would we admit such an occurrence in our attempt to explain the real world if we do not allow inconsistency? Most theories have taken a stand against such an occurrence with an axiom that assumes transitivity and prohibits intransitivity, although one does not have to be intransitive to be inconsistent in the values obtained. Others have wished it away by saying that it should not happen in human thinking. But it does, and we offer a theory below to deal with it.

## 2) The Fundamental Scale of the AHP for Making Comparisons with Judgments

If we were to use judgments instead of ratios, we would estimate the ratios as numbers using the Fundamental Scale of the AHP, shown in Table 1 and derived analytically later in the paper, and enter these judgments in the matrix. A judgment is made on a pair of elements with respect to a property they have in common. The smaller element is considered to be the unit and one estimates how many times more important, preferable or likely, more generally "dominant", the other is by using a number from the fundamental scale. Dominance is often interpreted as importance when comparing the criteria and as preference when comparing the alternatives with respect to the criteria. It can also be interpreted as likelihood as in the likelihood of a person getting elected as president, or other terms that fit the situation.

Table 1 The Fundamental Scale of Absolute Numbers

| Intensity of Importance | Definition | Explanation |
|---|---|---|
| 1 | Equal Importance | Two activities contribute equally to the objective |
| 2 | Weak or slight | |
| 3 | Moderate importance | Experience and judgment slightly favor one activity over another |
| 4 | Moderate plus | |
| 5 | Strong importance | Experience and judgment strongly favor one activity over another |
| 6 | Strong plus | |
| 7 | Very strong or demonstrated importance | An activity is favored very strongly over another; its dominance demonstrated in practice |

| | | |
|---|---|---|
| 8 | Very, very strong | |
| 9 | Extreme importance | The evidence favoring one activity over another is of the highest possible order of affirmation |
| Reciprocals of above | If activity i has one of the above nonzero numbers assigned to it when compared with activity j, then j has the reciprocal value when compared with i | A reasonable assumption |
| Rationals | Ratios arising from the scale | If consistency were to be forced by obtaining n numerical values to span the matrix |

The set of objects being pairwise compared must be homogeneous. That is, the dominance of the largest object must be no more than 9 times the smallest one (this is the widest span we use for many good reasons discussed elsewhere in the AHP literature). Things that differ by more than this range can be clustered into homogeneous groups and dealt with by using this scale. If measurements from an existing scale are used, they can simply be normalized without regard to homogeneity. When the elements being compared are very close, they should be compared with other more contrasting elements, and the larger of the two should be favored a little in the judgments over the smaller. We have found this approach to be effective to bring out the actual priorities of the two close elements. Otherwise we have proposed the use of a scale between 1 and 2 using decimals and similar judgments to the Fundamental Scale above. We note that human judgment is relatively insensitive to such small decimal changes.

Table 2 shows how an audience of about 30 people, using consensus to arrive at each judgment, provided judgments to estimate the *dominance* of the consumption of drinks in the United States (which drink is consumed more in the US and how much more than another drink?).

The derived vector of relative consumption and the actual vector, obtained by normalizing the consumption given in official statistical data sources, are at the bottom of the table.

Table 2. Relative Consumption of Drinks

**Which Drink is Consumed More in the U.S.?**
**An Example of Estimation Using Judgments**

| Drink Consumption in the U.S. | Coffee | Wine | Tea | Beer | Sodas | Milk | Water |
|---|---|---|---|---|---|---|---|
| **Coffee** | 1 | 9 | 5 | 2 | 1 | 1 | 1/2 |
| **Wine** | 1/9 | 1 | 1/3 | 1/9 | 1/9 | 1/9 | 1/9 |
| **Tea** | 1/5 | 2 | 1 | 1/3 | 1/4 | 1/3 | 1/9 |
| **Beer** | 1/2 | 9 | 3 | 1 | 1/2 | 1 | 1/3 |
| **Sodas** | 1 | 9 | 4 | 2 | 1 | 2 | 1/2 |
| **Milk** | 1 | 9 | 3 | 1 | 1/2 | 1 | 1/3 |
| **Water** | 2 | 9 | 9 | 3 | 2 | 3 | 1 |

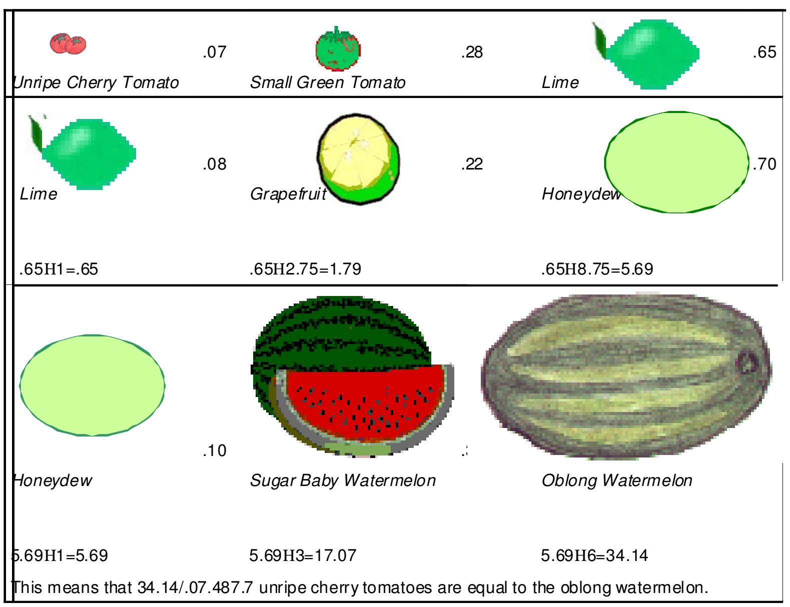
The derived scale based on the judgments in the matrix is:

| Coffee | Wine | Tea | Beer | Sodas | Milk | Water |
|---|---|---|---|---|---|---|
| .177 | .019 | .042 | .116 | .190 | .129 | .327 |

with a consistency ratio of .022.

The actual consumption (from statistical sources) is:

| | | | | | | |
|---|---|---|---|---|---|---|
| .180 | .010 | .040 | .120 | .180 | .140 | .330 |




If the objects are not homogenous they may be divided into groups that are homogeneous. If necessary additional objects can be added merely to fill out the intervening clusters to move from the smallest object to the largest one. Figure 3 shows how this process works in comparing a cherry tomato with a water melon, which appears to be two orders of magnitude bigger in size, by introducing intermediate objects in stages.

### 2) Scales of Measurement

Mathematically a scale is a triple, a set of numbers, a set of objects and a mapping of the objects to the numbers. There are two ways to perform measurement, one is by using an instrument and making the correspondence direct, and the other is by using judgment. When using judgments one can either assign numbers to the objects by guessing their value on some scale of measurement when there is one, or derive a scale by considering a subset of objects in some fashion such as comparing them in pairs, thus making the correspondence indirect. In addition there are two kinds of origin; one is an absolute origin as in absolute temperature where nothing falls below that reading; and the other where the origin is a dividing point of positive and negative values with no bound on either side such as with a thermometer. Underlying both these ways are the following kinds (there can be more) of general scales:

**Nominal Scale** invariant under one to one correspondence where a number is assigned to each object; for example, handing out numbers for order of service to people in a queue.

**Ordinal Scale** invariant under monotone transformations, where things are ordered by number but the magnitudes of the numbers only serve to designate order, increasing or decreasing; for example, assigning two numbers 1 and 2, to two people to indicate that one is taller than the other, without including any information about their actual heights. The smaller number may be assigned to the taller person and vice versa.

**Interval Scale** invariant under a positive linear transformation; for example, the linear transformation F = (9/5) C + 32 for converting a Celsius to a Fahrenheit temperature reading. Note that one cannot add two readings $x_1$ and $x_2$ on an interval scale because then $y_1 + y_2 = (a\,x_1 + b) + (a\,x_2 + b) = a\,(x_1 + x_2) + 2b$ which is of the form $ax + 2b$ and not of the form $ax + b$. However, one can take an average of such readings because dividing by 2 yields the correct form.

**Ratio Scale** invariant under a similarity transformation, $y = ax$, $a > 0$. An example is converting weight measured in pounds to kilograms by using the similarity transformation $K = 2.2\,P$. The ratio of the weights of the two objects is the same regardless of whether the measurements are done in pounds or in kilograms. Zero is not the measurement of anything; it applies to objects that do not have the property and in addition one cannot divide by zero to preserve ratios in a meaningful way. Note that one can add two readings from a ratio scale, but not multiply them because $a^2 x_1 x_2$ does not have the form $ax$. The ratio of two readings from a ratio scale such as 6 kg/ 3 kg = 2 is a number that belongs to an absolute scale that says that the 6 kg object is twice heavier than the 3 kg object. The ratio 2 cannot be changed by some formula to another number. Thus we introduce the next scale.

**Absolute Scale**: invariant under the identity transformation $x = x$; for example, numbers used in counting the people in a room.

There are also other less well-known scales like a logarithmic and a log-normal scale.

The fundamental scale of the AHP is a scale of absolute numbers used to answer the basic question in all pairwise comparisons: **how many times more dominant is one element than the other with respect to a certain criterion or attribute?** The derived scale, obtained by solving a system of homogeneous linear equations whose coefficients are absolute numbers, is also an absolute scale of relative numbers. Such a relative scale does not have a unit nor does it have an absolute zero. The derived scale is like probabilities in not having a unit or an absolute zero.

In a judgment matrix $A$, instead of assigning two numbers $w_i$ and $w_j$ (that generally we do not know), as one does with tangibles, and forming the ratio $w_i / w_j$ we assign a single number drawn from the fundamental scale of absolute numbers shown in Table 1 above to represent the ratio $(w_i / w_j)/1$. It is a nearest integer approximation to the ratio $w_i / w_j$. The ratio of two numbers from a ratio scale (invariant under multiplication by a positive constant) is an absolute number (invariant under the identity transformation) and is dimensionless. In other words it is not measured on a scale with a unit starting from zero. The numbers of an absolute scale are defined in terms of similarity or equivalence. The (absolute) number of a class is the class of all those classes that are similar to it; that is they can be put into one-to-one correspondence with it. But that is not our complete story about absolute numbers transformed to relative form-relative absolute numbers. We now continue our account.

The derived scale will reveal what $w_i$ and $w_j$ are. This is a central fact about the relative measurement approach. It needs a fundamental scale to express numerically the relative dominance relationship by using the smaller or lesser element as the unit of each comparison. Some people who do not understand this and regard the AHP as controversial, forget that most people in the world don't think in terms of numbers but of how they feel about intensities of dominance. They think that the AHP would have a greater theoretical strength if the judgments were made in terms of "ratios of preference differences". I think that the layman would find this proposal laughable as I do for its paucity of understanding, taking the difference of non-existing numbers which one is trying to find in the first place. He needs first to see a utility doctor who would help him create an interval scale utility function so he can take values from it to form differences and then form their ratios to get one judgment!

### 3) From Consistency to Inconsistency

Consistency is essential in human thinking because it enables us to order the world according to dominance. It is a necessary condition for thinking about the world in a scientific way, but it is not sufficient because a mentally disturbed person can think in a perfectly consistent way about a world that does not exist. We need actual knowledge about the world to validate our thinking. But if we were always consistent we would not be able to change our minds. New knowledge often requires that we see things in a new light that can contradict what we thought was correct before. Thus we live with the contradiction that we must be consistent to capture valid knowledge about the world but at the same time be ready to change our minds and be inconsistent if new information requires that we think differently than we thought before. It is clear that large inconsistency unsettles our thinking and thus we need to change our minds in small steps to integrate new information in the old total scheme. This means that inconsistency must be large enough to allow for change in our consistent understanding but small enough to make it possible to adapt our old beliefs to new information. *This means that inconsistency must be precisely one order of magnitude less important than consistency, or simply 10% of the total concern with consistent measurement. If it were larger it would disrupt consistent measurement and if it were smaller it would make insignificant contribution to change in measurement*.

The paired comparisons process using actual measurements for the elements being compared leads to the following consistent reciprocal matrix:

$$\begin{array}{cc} & \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \\ w_1 & w_2 & \cdots & w_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} & \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \cdots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix} \end{array}$$

We note that we can recover the vector $w = (w_1, ..., w_n)$ by solving the system of equations defined by:

$$Aw = \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \cdots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = nw$$

Solving this homogeneous system of linear equations $Aw = nw$ to find w is a trivial eigenvalue problem, because the existence of a solution depends on whether or not *n* is an eigenvalue of the characteristic equation of *A*. But *A* has rank one and thus all its eigenvalues but one are equal to zero. The sum of the eigenvalues of a matrix is equal to its trace, the sum of its diagonal elements, which in this case is equal to *n*. Thus *n* is the largest or the principal eigenvalue of *A* and *w* is its corresponding principal eigenvector that is positive and unique to within multiplication by a constant, and thus belongs to a ratio scale. We now know what must be done to recover the weights $w_i$, whether they are known in advance or not.

We said earlier that an *n* by *n* matrix $A = (a_{ij})$ is consistent if $a_{ij}a_{jk} = a_{ik}, i, j, k = 1,...,n$ holds among its entries. We have for a consistent matrix $A^k = n^{k-1}A$, a constant times the original matrix. In normalized form both *A* and $A^k$ have the same principal eigenvector. That is not so for an inconsistent matrix. A consistent matrix always has the form $A = (\frac{w_i}{w_j})$. Of course, real- world pairwise comparison matrices are very unlikely to be consistent.

Later we derive priorities for the inconsistent case through dominance arguments. Now we give an elegant mathematical discussion to show why we still need for an inconsistent matrix the principal right eigenvector for our priority vector. It is clear that no matter what method we use to derive the weights $w_i$, we need to get them back as proportional to the expression $\sum_{j=1}^{n} a_{ij} w_j \quad i = 1,...,n$, that is, we must solve $\sum_{j=1}^{n} a_{ij} w_j = cw_i \quad i = 1,...,n$. Otherwise $\sum_{j=1}^{n} a_{ij} w_j \quad i = 1,...,n$ would yield another set of different weights and they in turn can be used to form new expressions $\sum_{j=1}^{n} a_{ij} w_j \quad i = 1,...,n$, and so on ad infinitum. Unless we solve the principal eigenvalue problem, our quest for priorities becomes meaningless.

We learn from the consistent case that what we get on the right is proportional to the sum on the left that involves the same ratio scale used to weight the judgments that we are looking for. Thus we have the proportionality constant c. A better way to see this is to use the derived vector of priorities to weight each row of the matrix and take the sum. This yields a new vector of priorities (relative dominance of each element) represented in the comparisons. This vector can again be used to weight the rows and obtain still another vector of priorities. In the limit (if one exists), the limit vector itself can be used to weight the rows and get the limit vector back perhaps proportionately. Our general problem possibly with inconsistent judgments takes the form:

$$Aw = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = cw$$

This homogeneous system of linear equations $Aw = cw$ has a solution *w* if *c* is the principal eigenvalue of $A$. That this is the case can be shown using an argument that involves both left and right eigenvectors of *A*. Two vectors $x = (x_1,...,x_n),\ y = (y_1,...,y_n)$ are orthogonal if their scalar product $x_1 y_1 + ... + x_n y_n$ is equal to zero. It is known that any left eigenvector of a matrix corresponding to an eigenvalue is orthogonal to any right eigenvector corresponding to a different eigenvalue. This property is known as biorthogonality [3].

**Theorem** for a given positive matrix *A*, the only positive vector *w* and only positive constant *c* that satisfy $Aw = cw$, is a vector *w* that is a positive multiple of the principal eigenvector of *A*, and the only such c is the principal eigenvalue of *A*.

## 4) An Example of an AHP Decision

The simple decision is to choose the best city in which to live. We shall show how to make this decision using both methods of the AHP which conform with what Blumenthal said. We do it first with relative (comparative) measurement and second with absolute measurement. With the relative measurement method the criteria are pairwise compared with respect to the goal, the alternatives are pairwise compared with respect to each criterion and the results are synthesized or combined using a weighting and adding process to give an overall ranking of the alternatives. With the absolute measurement method standards are established for each criterion and the cities are rated one-by-one against the standards rather than being compared with each other.Proof: We know that the right principal eigenvector and the principal eigenvalue satisfy our requirements. We also know that the algebraic multiplicity of the principal eigenvalue is one, and that there is a positive left eigenvector of $A$ (call it $z$) corresponding to the principal eigenvalue. Suppose there is a positive vector $y$ and a (necessarily positive) scalar $d$ such that $Ay = dy$ . If $d$ and $c$ are not equal, then by biorthogonality $y$ is orthogonal to $z$, which is impossible since both vectors are positive. If $c$ and $d$ are equal, then $y$ and $w$ are dependent since c has algebraic multiplicity one, and $y$ is a positive multiple of $w$. This completes the proof.

## 6. Making the Decision with a Relative Measurement Model

The model for choosing the best city in which to live is shown below in Figure 3. It has a goal at the top, criteria in the middle and cities at the bottom. We need to prioritize the criteria in terms of the goal and the cities in terms of the criteria.

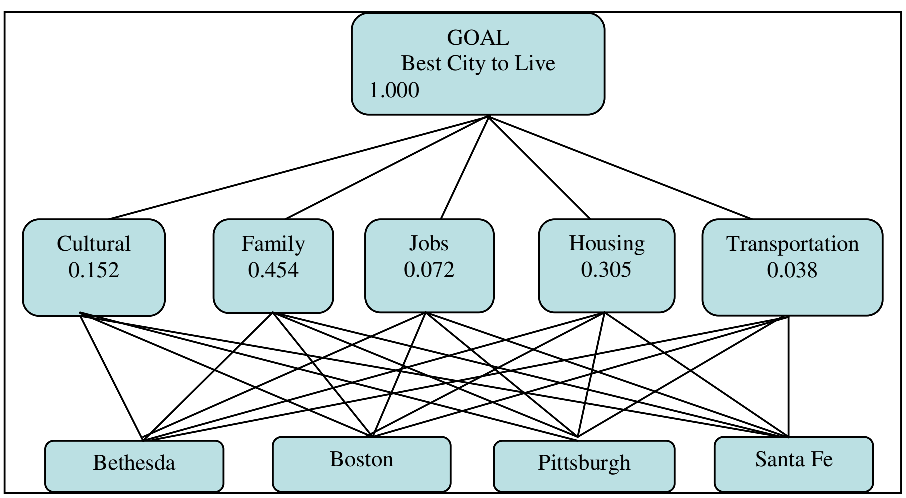



Figure 3. Relative Model for Choosing Best City to Live in.

## Entering Judgments

For each cell in the comparison matrix there is associated a row criterion (listed on the left), call it X, and a column criterion (on the top), call it Y. One answers this question for the cell: How much more important is X than Y in choosing a best city in which to live? The judgments, shown in

Table 2, are entered using the fundamental scale of the AHP. Fractional values between the integers such as 4.32 can also be used when they are known from measurement.

## The Number of Judgments and Consistency

In Table 3 with its comparisons of the relative importance of the five criteria with respect to the goal in Figure 4, there are 10 judgments to be entered. As we shall see later, inconsistency for a judgment matrix can be computed as a function of its maximum eigenvalue $\lambda_{\max}$ and the order $n$ of the matrix. The time gained, from making fewer judgments than 10 along a spanning tree for example can be offset by not having sufficient redundancy in the judgments to fine tune and improve the overall outcome. There can be no inconsistency when the minimum number of judgments is used.

Table 2. Criteria Weights with Respect to the Goal

| GOAL | Culture | Family | Housing | Jobs | Transportation | Priorities |
|---|---|---|---|---|---|---|
| Culture | 1 | 1/5 | 3 | 1/2 | 5 | 0.152 |
| Family | 5 | 1 | 7 | 1 | 7 | 0.433 |
| Housing | 1/3 | 1/7 | 1 | 1/4 | 3 | 0.072 |
| Job | 2 | 1 | 4 | 1 | 7 | 0.305 |
| Transportation | 1/5 | 1/7 | 1/3 | 1/7 | 1 | 0.038 |

Inconsistency 0.05

Next the alternatives are pairwise compared with respect to each of the criteria. The judgments and the derived priorities for the alternatives are shown in Table 4. The priority vectors are the principal eigenvectors of the pairwise comparison matrices. They are in the distributive form, that is, they have been normalized by dividing each element of the principal eigenvector by the sum of its elements so that they sum to 1. The priority vectors can transformed to their idealized form by selecting the largest element in the vector and dividing all the elements by it so that it takes on the value 1, with the others proportionately less. The element (or elements) with a priority of 1 become the ideal(s). Later we explain why we use these two forms of synthesis.

### Synthesis

The outcome of the distributive form is shown in Table 4 and that for the ideal form is shown in Table 5. The columns in Table 4 are the priority vectors for the cities from Table 4 and the columns in Table 5 are these same vectors in idealized form with respect to each criterion. Using either form the totals vector is obtained by multiplying the priority of each criterion times the priority of each alternative with respect to it and summing. The overall priority vector is obtained from the totals vector by normalizing: dividing each element in the totals vector by the sum of its elements. The final outcome with either form of synthesis is that Pittsburgh is the highest ranked city for this individual. Though the final priorities are somewhat different the order is the same: Pittsburgh, Boston, Bethesda and Santa Fe. The ratios of the final priorities are meaningful. Pittsburgh is almost twice as preferred as Bethesda.

When synthesizing in the distributive form the totals vector and the overall priorities vector are the same. When synthesizing in the ideal form as shown in Table 5 they are not. Ideal synthesis gives slightly different results from distributive synthesis in this case.

Table 3: Alternatives' Weights with Respect to Criteria

| Culture | Bethesda | Boston | Pittsburgh | Santa Fe | Priorities |
|---|---|---|---|---|---|
| Bethesda | 1 | 1/2 | 1 | 1/2 | 0.163 |
| Boston | 2 | 1 | 2.5 | 1 | 0.345 |
| Pittsburgh | 1 | 1/2.5 | 1 | 1/2.5 | 0.146 |
| Santa Fe | 2 | 1 | 2.5 | 1 | 0.345 |

Inconsistency .002

| Family | Bethesda | Boston | Pittsburgh | Santa Fe | Priorities |
|---|---|---|---|---|---|
| Bethesda | 1 | 2 | 1/3 | 4 | 0.210 |
| Boston | 1 | 1 | 1/8 | 2 | 0.098 |
| Pittsburgh | 3 | 8 | 1 | 9 | 0.635 |
| Santa Fe | 1/4 | 1/2 | 1/9 | 1 | 0.057 |

Inconsistency .012

| Housing | Bethesda | Boston | Pittsburgh | Santa Fe | Priorities |
|---|---|---|---|---|---|
| Bethesda | 1 | 5 | 1/2 | 2.5 | 0.262 |
| Boston | 1/5 | 1 | 1/9 | 1/4 | 0.047 |
| Pittsburgh | 2 | 9 | 1 | 7 | 0.571 |
| Santa Fe | 1/2.5 | 4 | 1/7 | 1 | 0.120 |

Inconsistency .012

| Jobs | Bethesda | Boston | Pittsburgh | Santa Fe | Priorities |
|---|---|---|---|---|---|
| Bethesda | 1 | 1/2 | 3 | 4 | 0.279 |
| Boston | 2 | 1 | 6 | 8 | 0.559 |
| Pittsburgh | 1/3 | 1/6 | 1 | 1 | 0.087 |
| Santa Fe | 1/4 | 1/8 | 1 | 1 | 0.075 |

Inconsistency .004

| Transportation | Bethesda | Boston | Pittsburgh | Santa Fe | Priorities |
|---|---|---|---|---|---|
| Bethesda | 1 | 1.5 | 1/2 | 4 | 0.249 |
| Boston | 1/1.5 | 1 | 1/3.5 | 2.5 | 0.157 |
| Pittsburgh | 2 | 3.5 | 1 | 9 | 0.533 |
| Santa Fe | 1/4 | 1/2.5 | 1/9 | 1 | 0.061 |

Inconsistency .001

Table 4. Synthesis using the Distributive Mode to Obtain the Overall Priorities for the Alternatives

| **Synthesis** | **Cultural** ***0.152*** | **Family** ***0.433*** | **Housing** ***0.072*** | **Jobs** ***0.305*** | **Transport** ***0.038*** | **Totals** (Weight and add) | **Overall Priorities** (Normalize Totals) |
|---|---|---|---|---|---|---|---|
| **Bethesda** | 0.163 | 0.210 | 0.262 | 0.279 | 0.249 | 0.229 | 0.229 |
| **Boston** | 0.345 | 0.098 | 0.047 | 0.559 | 0.157 | 0.275 | 0.275 |
| **Pittsburgh** | 0.146 | 0.635 | 0.571 | 0.087 | 0.533 | 0.385 | 0.385 |
| **Santa Fe** | 0.345 | 0.057 | 0.120 | 0.075 | 0.061 | 0.111 | 0.111 |

Table 5. Synthesis using the Ideal Mode to Obtain the Overall Priorities for the Alternatives

| **Synthesis** | **Cultural** ***0.152*** | **Family** ***0.433*** | **Housing** ***0.072*** | **Jobs** ***0.305*** | **Transport** ***0.038*** | **Totals** (Weight and add) | **Overall Priorities** (Normalize Totals) |
|---|---|---|---|---|---|---|---|
| **Bethesda** | 0.474 | 0.330 | 0.459 | 0.500 | 0.467 | 0.418 | 0.224 |
| **Boston** | 1.000 | 0.155 | 0.082 | 1.000 | 0.295 | 0.541 | 0.290 |
| **Pittsburgh** | 0.424 | 1.000 | 1.000 | 0.155 | 1.000 | 0.655 | 0.351 |
| **Santa Fe** | 1.000 | 0.089 | 0.209 | 0.135 | 0.115 | 0.251 | 0.135 |

### Ideal Synthesis Prevents Rank Reversal

An important distinction to make between measurement in physics and measurement in decision making is that in the first we usually seek measurements that approximate to the weight and length of things, whereas in human action we seek to order actions according to priorities. In mathematics a distinction is made between *metric topology* that deals with the measurement of length, mass and time and *order topology* that deals with the ordering of priorities through the concept of *dominance* rather than closeness used in metric methods. We have seen that the principal eigenvector of a matrix is necessary to capture dominance priorities. When we have a matrix of judgments we derive its priorities in the form of its principal eigenvector. When we deal with a hierarchy the principle of hierarchic composition involves weighting and adding as a special case of the more general principle of network composition in which priorities are also derived as the principal eigenvector of a stochastic matrix which involves weighting and adding in the process of raising a matrix to powers. Some scholars whose specialization is in the physical sciences are perhaps unaware of the methods of order topology and have used various arguments to justify why they would use a metric approach to derive priorities and also to obtain the overall synthesis. It may be worthwhile to discuss this at some length in the following paragraph.

Ideal synthesis should be used when one wishes to prevent reversals in rank of the original set of alternatives from occurring when a new dominated alternative is added. With the distributive form rank reversal can occur to account for the presence of many other alternatives in cases where adding many things of the same kind or of nearly the same kind can depreciate the value of any of them. It has been established that 92% of the time, there is no rank reversal in the distributive mode when a new dominated alternative is added [16]. We note that uniqueness or manyness are not criteria that can be included when the alternatives are assumed to be independent of one another, for then to rank an alternative one would have to see how many other alternatives there are thus creating dependence among them.

Both the distributive and ideal modes are necessary for use in the AHP. We have shown that idealization is essential and is independent of what method one may use. There are people who have made it an obsession to find ways to avoid rank reversal in every decision and wish to alter the synthesis of the AHP away from normalization or idealization. They are likely to obtain outcomes that are not compatible with what the real outcome of a decision should be, because in decision-making we also want uniqueness of the answer we get.

Here is a failed attempt by some people to do things their metric way to preserve rank other than by the ideal form. The multiplicative approach to the

AHP uses the familiar methods of taking the geometric mean to obtain the priorities of the alternatives for each criterion without normalization, and then raising them to the powers of the criteria and again taking the geometric mean to perform synthesis in a distorted way to always preserve rank. It is essentially a consequence of attempting to minimize the logarithmic least squares expression $\sum_{i=1}^{n}\sum_{j=1}^{n}(\log a_{ij} - \log \frac{w_i}{w_j})^2$ . It does not work when the same measurement is used for the alternatives with respect to several criteria as one can easily verify and that should be sufficient to throw it out. Second and more seriously, the multiplicative method has an untenable mathematical problem. Assume that an alternative has a priority 0.2 with respect to each of two criteria whose respective priorities are 0.3 and 0.5. It is logical to assume that this alternative should have a higher priority with respect to the more important criterion, the one with the value of 0.5, after the weighting is performed. But $0.2^{0.5} < 0.2^{0.3}$ and alas it does not, it has a smaller priority. One would think that the procedure of ranking in this way would have been abandoned at first knowledge of this observation.

We conclude that in order to preserve rank indiscriminately from any other alternative, one can use the rating approach of the AHP described below in which alternatives are evaluated one at a time using the ideal mode. In addition, if in performing paired comparisons of the alternatives as in relative measurement, one wishes to preserve change in rank when irrelevant alternatives are introduced, that is, alternatives whose priorities are low under all the criteria, one can also use the ideal mode for synthesis.

Remark: On occasion someone has suggested the use of Pareto optimality instead of weighting the priorities of the alternatives by the priorities of the criteria and adding to find the best alternative. It is known that a concave function for the synthesis, if one could be found, would serve the purpose of finding the best alternative when it is known what it should be. But if the best alternative is already known for some property that it has which makes it the best, then one has a single not a multiple criterion decision. Naturally a multiple criterion problem may not yield the expected outcome. This is a special case of when the weights of the criteria depend on those of the alternatives. We will see in Part 2 that the final overall choice is automatically made in the process of finding the priorities of the criteria as they depend on the alternatives. Pareto optimality plays no role to determine the best outcome in that general case.

## 5. Making the Decision with an Absolute or Ratings Model

Using the absolute or ratings method of the AHP categories (intensities) or standards are established for the criteria and cities are rated one at a time by selecting the appropriate category under each criterion rather than compared against other cities. The standards are prioritized for each criterion by making pairwise comparisons. For example, the standards for the criterion Job Opportunities are: Excellent, Above Average, Average, Below Average and Poor. Judgments are entered for such questions as: "How much more preferable is Excellent than Above Average for this criterion? Each city is then rated by selecting the appropriate category for it for each criterion. The city's score is then computed by weighting the priority of the selected category by the priority of the criterion and summing for all the criteria. The prioritized categories are essentially absolute scales, abstract yardsticks, which have been derived and are unique to each criterion. Judgment is still required to select the appropriate category under a criterion for a city, but the cities are no longer compared against each other. In absolute measurement, the cities are scored independently of each other. In relative measurement, there is dependence, as a city's performance depends on what other cities there are in the comparison group. Figure 5 and Tables 7, 8 and 9 represent what one does in the ratings or absolute measurement approach of the AHP. Table 7 illustrates the pairwise comparisons of the intensities under one criterion. The process must be repeated to compare the intensities for each of the other criteria. We caution that such intensities and their priorities are only appropriate for our given problem and should not be used with the same priorities for all criteria nor carelessly in other problems.

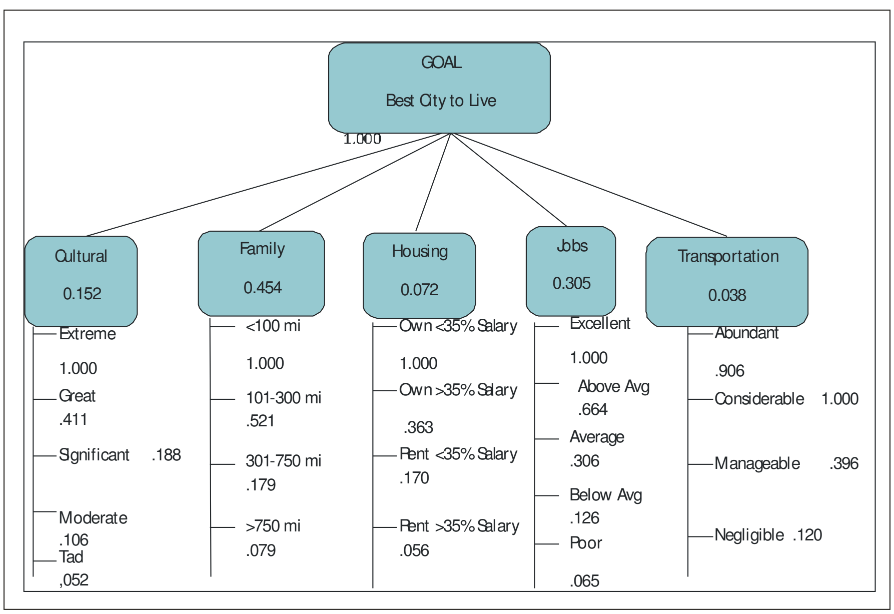



Figure 4. Absolute or Ratings Mode for Choosing a City to Live

Table 6. Deriving Priorities for the Cultural Criterion Categories

| | **Extreme** | **Great** | **Significant** | **Mode-rate** | **Tad** | *Derived Priorities* | *Idealized Priorities* |
|---|---|---|---|---|---|---|---|
| **Extreme** | 1 | 5 | 6 | 8 | **9** | .569 | 1.000 |
| **Great** | 1/5 | 1 | 4 | 5 | **7** | .234 | .411 |
| **Significant** | 1/6 | 1/4 | 1 | 3 | 5 | .107 | .188 |
| **Moderate** | 1/8 | 1/5 | 1/3 | 1 | 4 | .060 | .106 |
| **Tad** | 1/9 | 1/7 | 1/5 | 1/4 | 1 | .030 | .052 |

*Inconsistency = .112*

Table 7. Verbal Ratings of Cities under each Criterion

| Alternatives | **Cultural** *.195* | **Family** *.394* | **Housing** *.056* | **Jobs** *.325* | **Transport** *.030* | **Total Score** | **Priorities (Normal ized)** |
|---|---|---|---|---|---|---|---|
| **Pittsburgh** | Signific. | <100 mi | Own>35% | Average | Manageable | **.562** | **.294** |
| **Boston** | Extreme | 301-750 mi | Rent>35% | Above Avg | Abundant | **.512** | **.267** |
| **Bethesda** | Great | 101-300 mi | Rent<35% | Excel lent | Considerable | **.650** | **.339** |
| **Santa Fe** | Signific. | >750 mi | Own>35% | Average | Negligible | **.191** | **.100** |

Table 8. Priorities of Ratings of Cities under each Criterion

| Alternatives | **Cultural** *.195* | **Family** *.394* | **Housing** *.056* | **Jobs** *.325* | **Transport** *.030* | **Total Score** | **Priorities (Normal ized)** |
|---|---|---|---|---|---|---|---|
| **Pittsburgh** | 0.188 | 1.000 | 0.363 | 0.306 | 0.396 | **562** | **.294** |
| **Boston** | 1.000 | 0.179 | 0.056 | 0.664 | 0.906 | **512** | **.267** |
| **Bethesda** | 0.411 | 0.521 | 0.170 | 1.000 | 1.000 | **650** | **.339** |
| **Santa Fe** | 0.188 | 0.079 | 0.363 | 0.306 | 0.120 | **191** | **.100** |

## 6. Applications in Economics

We give three different kinds of applications of the AHP in economics. The first is concernd with exchanging the dollar and the Japanese yen. The second involves using judgments to derive an input-output table. The third is about forecasting the future trun around of the US economcy in 1992, 2001 and in 2008-2009. Having illustrated how the judgment process is used to make pairwise comparisons, we will ony illustrate the structures used to deal with these problems along with some useful comments in each example.

## THE FOREIGN EXCHANGE FORECASTING FRAMEWORK [2]

This part outlines a set of foreign exchange rate forecasting factors. The treatment is eclectic and draws upon existing theories of foreign exchange rate determination. It should be emphasized, however. that the identification and conceptualization of the factors may well vary depending on the composition of the expert group. Accordingly, application of this method requires that careful consideration be given to the range of competencies and experiences of the experts participating in the process.. However, small perturbations in the choices have been shwon to have little effect on the judgments. In each level below the goal we made the comparisons of the elements as to their relative importance with respect to each parent factor in the level immediately above. For the alternatives we ask which is the more likely outcome in comparing them with respect to the factor in the next to the last or bottom level.To obtain the final outcome, we multiplied the priority of each alternative by the midpoint of the interval of yen values corresponding to it and added thus computing the expected value. To estimate the judgments used in the comparisons in each matrix, one can divide the corresponding priorities of the elements being compared under a given factor above them. The reader interested in the process will have to enlarge the figure to determine the names and priorities of the elements all summarized conveniently in one figure that occupies a half a page.

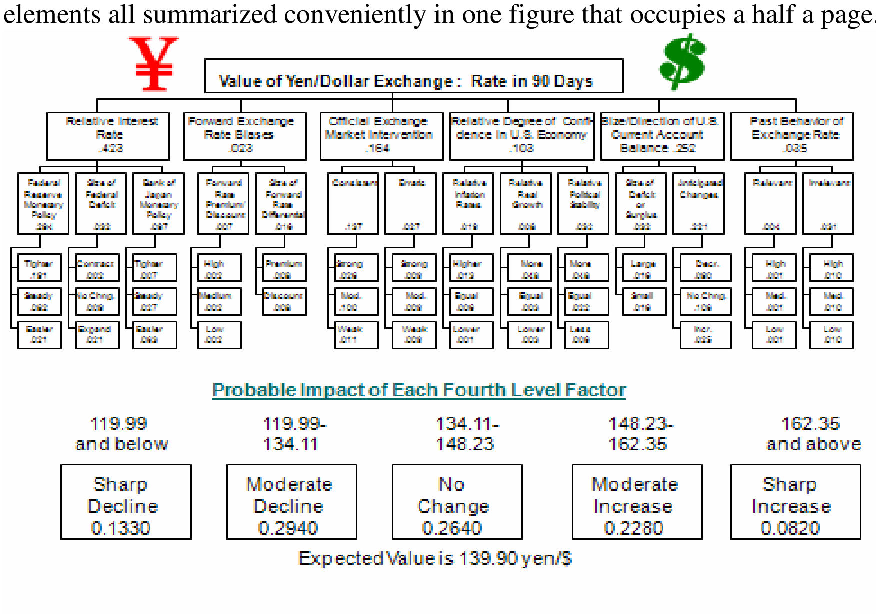



Figure 6 The Dollar versus the Yen

**ESTIMATION OF INPUT_OUTPUT TECNOLOGICAL COEFFICIENTS [3]**

The Sudan is the largest country I n area in Africa and is one of the most fertile countries in the world for growing food. It has been estimated that the Sudan could feed hundreds of millions of people in Africa. The study of the Sudanese economy in the 1970's in order to develop a plan for the transport sector at a macro level required the development of an econometric model based on an input-output approach done by the Nobel Laureate Lawrence Klein. This was done by using traditional methods. In this part we present a new systems-oriented method for estimating the input-output coefficients of an economy. A major advantage is that it generally does not require extensive detail to capture the significant relations among the sectors. Also, when desired intangibles coefficients may be included in the input-output model.

Often one's general understanding about an economy can be an adequate tool for doing this without the use of an enormous amount of statistical data. Also, the process does not require knowledge of valuation of outputs, secondary products, dummy industries, imports, inventory change, and gross inputs and outputs, which are usually needed and often prove to be problematic in input-output analysis.

Input-output models emerged through a process of historical development. In 1758, F. Quesnay published "Tableau Economique," in which he emphasized the importance of the interdependence among economic activities. Later he developed a modified "macroeconomic" version of the tableau, which represented the entire economy of his day in the form of circular flows.

In 1874, L. Walras, in his "Elements d'Economie Politique Pure," examined the simultaneous computation of all prices in an economy. His model consisted of a system of equations, one for each price. He was also interested in the general equilibrium of production. In his theory he made use of the coefficients of production, which were determined by the technology and measured the amount of each factor needed to produce one unit of each kind of finished product. The model developed by Walras considers interdependence among the producing sectors of the economy, together with the competing demand of each sector for the factors of production.

In 1936, W. Leontief developed a general theory of production based on the notion of economic interdependence. Leontief's input-output analysis is an adaptation of the theory of general equilibrium to the study of the quantitative interdependence among related economic activities. It was first formulated to analyze and measure the connection between the producing and the consuming sectors within a national economy, but it has also been applied to the economies of a region, of several regions of metropolitan areas and of the entire world. In all applications, whether the system is small or large, the interdependence among its sectors is described by a set of linear equations; its structural characteristics are expressed by means of the numerical magnitude of the coefficients of these equations. If we denote by $y_i$ the quantity that sector i allocates to the final demand sector, and by $x_i$ the total output of sector i, then the system of equations that provides the general equilibrium point of the economy is given by: $\sum_{j=1}^{n} a_{ij}x_j + y_i = x_i, \; i = 1, 2, ..., n.$ Then the solution of this system of equations in matrix form is given by: $x = (I - A)^{-1} y.$ Thus, once the coefficients of the system of equations are known, it is easy to compute the inverse of the matrix $(I - A)$. Our method estimates the coefficients $a_{ij}$ in a simpler manner than is currently done, and hence computing the equilibrium point of an economy is considerably simplified.

Application of the AHP to generate input-output coefficients proceeds in two steps. The first utilizes judgments to determine the relative impact of the different sectors on the economy. This is essentially an "a priori" or constant value of the sectors. The second step involves analysis of the interdependence among the sectors. We take each sector and determine the relative strength of utilization of its output by the remaining sectors. The second step is a calculation of the current value of a sector in terms of its influence on the remaining sectors. Finally we compose the results of the two steps to obtain the input-output matrix. A portion of the Sudan transport study involved the construction of econometric models with an input-output table by our colleague L.R. Klein at the Wharton Economic Forecasting Associates. This particular input-output table was developed on the basis of information from surrounding countries and not directly from Sudanese data. Thus it is an indirect estimate. We used our, procedure to obtain an input coefficients table based on qualitative information on the economic sectors of the Sudan. Since the number of sectors is small, it serves as a good, short illustration of the method. Similar applications have been made with respect to Pakistan and Iran. The approach we followed was to have my student at that time, now Professor Luis Vargas, who initially had no knowledge about the Sudan, examined the literature regarding interaction among the sectors of the economy in the Sudan. Only judgments of dominance of one activity over another were surmised from the reading. No numbers were studied or used, only judgments formed. We then used the AHP to construct the input-output matrix.

The sectors of the economy of the Sudan were identified by using a Sudan Transport Study done in 1975, and their interactions were noted (see Table 10).

Table 10 Pairwise Interactions of Economic Sectors

| | 1 AGR | 2 PU | 3 M&M | 4 T&D | 5 CONS | 6 SERV |
|---|---|---|---|---|---|---|
| 1. Agriculture | X | | X | X | X | |
| 2. Public utilities | X | | X | X | | X |
| 3. Manufacturing & mining | X | | | X | X | X |
| 4. Transportation & distribution | X | X | X | | X | X |
| 5. Construction | | | | | | X |
| 6. Services | X | X | X | X | X | |

From three studies it was determined that agriculture and transportation and distribution were comparable, but that for the sake of comparing homogeneous elements as in the AHP, the remaining sectors had to be clustered to be in a class comparable to these two. Thus we have:

$$\text{Cluster}\begin{cases}\text{Public utilities}\\ \text{Manufacturing and mining}\\ \text{Construction}\\ \text{Services}\end{cases}$$

We also have the matrix of pairwise comparisons and principal eigenvector of weights shown in Tables 11-20.

Table 11 Sector contribution to economy

| | *AGR* | *T&D* | *Clustere* | *Weights* |
|---|---|---|---|---|
| AGR | 1 | 1/2 | 2 | 0.311 |
| T&D | 2 | 1 | 2 | 0.493 |
| Cluster | 1/2 | 1/2 | 1 | 0.195 |

All sectors in the cluster fall in the same comparability class; their pairwise comparison matrix according to their contribution to the economy, together with their corresponding eigenvector of weights, is given by:

Table 12 Other sector contribution to economy

*Contribution to Economy*

| | *P.U.* | *M&M* | *CONS* | *SERV* | *Weights* |
|---|---|---|---|---|---|
| P.U. | 1 | 1/2 | 1/2 | 1/3 | 0.127 |
| M&M | 2 | 1 | 1 | 1 | 0.280 |
| CONS | 2 | 1 | 1 | 1 | 0.280 |
| SERV | 3 | 1 | 1 | 1 | 0.312 |

The eigenvector above is then multiplied by the weight of the cluster, which is 0.195 from the previous eigenvector. Finally, we concatenate the weights of the six sectors and obtain for the index of relative importance of the sectors in the economy of the Sudan:

| *Sectors* | AGR | P.U. | M&M | T&D | CONS | SERV |
|---|---|---|---|---|---|---|
| Total index of relative importance | 0.3108 | 0.0248 | 0.0546 | 0.4934 | 0.0546 | 0.0608 |

The next step is to compare the sectors according to the contribution they receive from each of the six sectors.

Using information from the literature, we aggregate all but manufacturing and mining into a cluster:

$$\text{Cluster}\begin{cases}\text{Transportation and distribution}\\ \text{Agriculture}\\ \text{Construction}\end{cases}$$

and compare the two according to the contribution they receive from agriculture. We have:

| | *Cluster* | *M&M* | Weights |
|---|---|---|---|
| Cluster | 1 | 1/3 | 0.3333 |
| M&M | 3 | 1 | 0.6667 |

Manufacturing and mining is considered weakly superior to the cluster receiving inputs from the agricultural sector. The reason for this is that the main crop is cotton, which is allocated to manufacturing and exports.

Within the cluster, a subcluster consisting of agriculture and transportation and distribution is formed:

$$\text{Subcluster} \begin{cases} \text{Agriculture} \\ \text{Transportation and distribution} \end{cases}$$

The reason why these two sectors belong together is that the government itself makes most of the investment in agriculture and transportation. Thus the agricultural sector does not allocate much to itself and to transportation since they are preoccupations of the government. The activities within the subcluster are compared among themselves:

Table 14 More contribution from agriculture

| | *AGR* | *T&D* | *Weights* |
|---|---|---|---|
| AGR | 1 | 9 | 0.9 |
| T&D | 1/9 | 1 | 0.1 |

Agriculture receives by far the greater input in the form of earnings, seeds, and related materials than does the transportation sector because the outputs of the agricultural sector that are not exported or allocated to manufacturing and mining are used to grow new crops and to satisfy domestic consumption. It is natural to assume that private earnings of the agricultural sector are allocated to construction. If the subcluster is compared with construction, we have:

Table 15 Still more contribution from agriculture

| | *subcluster* | *CONS* | *Weights* |
|---|---|---|---|
| subcluster | 1 | 1/3 | 0.3333 |
| CONS | 3 | 1 | 0.6667 |

Hence construction is weakly superior to the subcluster because the government invests more in agriculture and transportation than in any other sector. What agriculture produces serves two objectives: (1) to satisfy internal needs and (2) to provide benefits for the people in agriculture. Since the industrial sector is the most rewarding sector for investing private earnings, and since it is not a significant part of the total GDP (3 percent), the only nongovernment-controlled sector remaining in which agriculture can allocate its outputs is the construction sector.

Composing the weights obtained for the cluster and subcluster, we obtain the priority weights for the sectors related to agriculture:

| *Sectors* | AGR | P.U. | M&M | T&D | CONS | SERV |
|---|---|---|---|---|---|---|
| Relative contribution from agriculture | 0.0225 | 0.0000 | 0.7500 | 0.0025 | 0.2250 | 0.0000 |

Similarly, we have the following matrices and eigenvectors for the contributions of the other five sectors.

Table 16 Contribution from public utilities

| | *AGR* | *M&M* | *T&D* | *SERV* | *Weights* |
|---|---|---|---|---|---|
| AGR | 1 | 1/9 | 1/7 | 1/5 | 0.0410 |
| M&M | 9 | 1 | 2 | 5 | 0.5242 |
| T&D | 7 | 1/2 | 1 | 3 | 0.3030 |
| SERV | 5 | 115 | 1/3 | 1 | 0.1318 |

Table 17 Contribution from transportation and distribution

| | *AGR* | *P.U.* | *M&M* | *T&D* | *SERV* | *Weights* |
|---|---|---|---|---|---|---|
| AGR | 1 | 1/3 | 1/2 | 1/2 | 7 | 0.0400 |
| P.U. | 3 | 1 | 1 | 2 | 9 | 0.3434 |
| M&M | 2 | 1 | 1 | 1 | 7 | 0.2596 |
| T&D | 2 | 1/2 | 1 | 1 | 7 | 0.2260 |
| SERV | 1/7 | 1/9 | 1/7 | 1/7 | 1 | 0.0310 |

The construction sector allocates outputs only to services. Thus the priority associated with services is 1.

For the contribution of the services sector we have a cluster consisting of:

$$\text{Cluster}\begin{cases}\text{Construction}\\ \text{Services}\end{cases}$$

for which we get:

Table 18 Contribution from services

| | *CONS* | *SERV* | *Weights* |
|---|---|---|---|
| CONS | 1 | 9 | 0.9 |
| SERV | 1/9 | 1 | 0.1 |

Construction receives most of what is allocated to the cluster.

With information from the government of Sudan and others we compare the remaining sectors, together with the cluster, and get:

Table 19 More contribution from services

| | *P.U.* | *M&M* | *T&D* | *Cluster* | *Weights* |
|---|---|---|---|---|---|
| P.U. | 1 | 1/2 | 1/2 | 3 | 0.1930 |
| M&M | 2 | 1 | 1 | 5 | 0.3680 |
| T&D | 3 | 1 | 1 | 5 | 0.3680 |
| Cluster | 1/3 | 1/5 | 1/5 | 1 | 0.0704 |

The weights of construction and services are obtained by multiplying 0.0704, the weight of the cluster, by 0.9 and 0.1, respectively. The contribution of services to the other sectors is given by:

| *Sectors* | *AGR* | *P.U* | *M&M* | *T&D* | *CONS* | *SERV* |
|---|---|---|---|---|---|---|
| Relative Contribution of services | 0.0000 | 0.1930 | 0.3680 | 0.3680 | 0.0634 | 0.0070 |

The eigenvector of weights obtained in the second step is used to form the *rows* of a matrix, with zeros in positions where no interaction was indicated in the matrix of interactions. They represent the distribution of the output produced by a sector to the sectors related to it. We now use the first entry of the first eigenvector we obtained for the relative importance of the sectors to weight each element of the first row of this matrix, the second entry to weight each element of the second row, and so on. This yields the estimate of the input-output matrix given in Table 20.

Table 20 Estimates of Input-Output Coefficients

| | *AGR* | *P.U.* | *M&M* | *T&D* | *CONS* | *SERV* |
|---|---|---|---|---|---|---|
| AGR | 0.0079 | 0 | 0.2331 | 0.0008 | 0.0699 | 0 |
| P.U. | 0.0009 | 0 | 0.0130 | 0.0075 | 0 | 0.0033 |
| M&M | 0.0041 | 0 | 0 | 0.0089 | 0.0379 | 0.0037 |
| T&D | 0.0691 | 0.1694 | 0.1281 | 0 | 0.1115 | 0.0153 |
| CONS | 0 | 0 | 0 | 0 | 0 | 0.0546 |
| SERV | 0 | 0.0117 | 0.0224 | 0.0224 | 0.0039 | 0.0004 |

Table 21. Actual Input-Output Coefficients Obtained in the Sudan Transport Study

| | *AGR* | *P. U.* | *M&M* | *T&D* | *CONS* | *SER V* |
|---|---|---|---|---|---|---|
| AGR | 0.00737 | 0 | 0.21953 | 0.00042 | 0.06721 | 0 |
| P.U. | 0.00024 | 0 | 0.01159 | 0.00618 | 0 | 0.00283 |
| M&M | 0.00393 | 0 | 0 | 0.00857 | 0.04216 | 0.00322 |
| T&D | 0.06993 | 0.14536 | 0.12574 | 0 | 0.09879 | 0.00641 |
| CONS | 0 | 0 | 0 | 0 | 0 | 0.05402 |
| SERV | 0 | 0.01030 | 0.02549 | 0.02422 | 0.00520 | 0.00021 |

Comparison with Table 21, constructed by the Wharton Econometric Forecasting Associates, which itself involved considerable approximation, shows that most differences between the elements of the two tables are small.

## FORECASTING TURNAROUND OF THE U.S ECONOMY, 1992 [4], 2001[5], 2008-2009 (in process)

Our forecasting exercises employed the AHP to address two critical issues germane to forecasting: the timing and the strength of the expected recovery. The timing issue required us to incorporate into the forecasting exercise the sequence of global events of the previous two and a half years. In our view these events had been forging a restructuring of global resources and institutional arrangements. With regard to the strength of the recovery, our task was to think through the ways in which such restructuring acts as a moderating influence on the performance of the key macroeconomic variables most proximately connected to the U.S. economic cycle. Our first exercise thus sought to forecast the most likely period for the turnaround, while the second tried to predict the strength of the ensuing recovery.

## DECOMPOSITION OF THE PROBLEM HIERARCHICALLY

As noted, the objective of the first exercise was to forecast the most likely date of a turnaround. The top level of both exercises consists of the factors representing the forces or major influences driving the economy. These forces are grouped into two categories: "conventional adjustment" and "economic restructuring." Both of these categories are decomposed into subfactors represented in the second level. For the timing forecast, the third level consists of time periods in which the recovery can occur. Figure 7 provides a schematic layout used to forecast the timing of the economic turnaround.

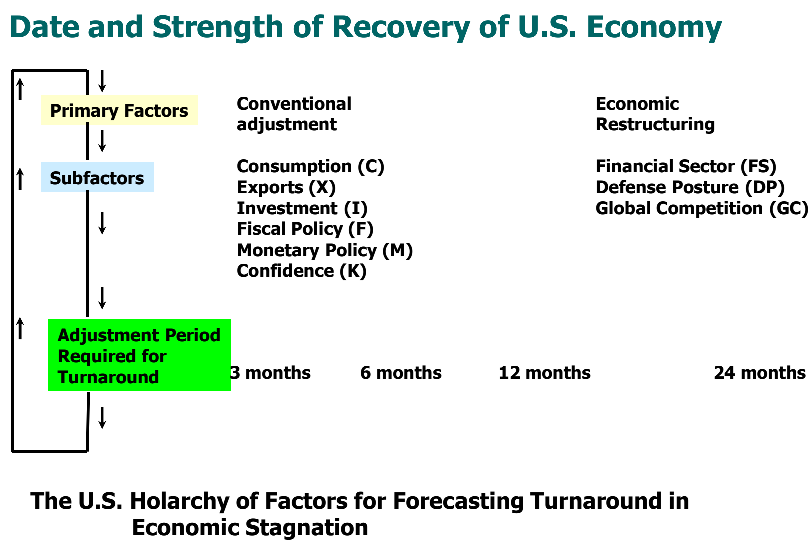



Figure 7 Factors and time periods that influenced the date of turnaround of the US economy in 1992

The judgments with regard to the identification of factors as well as the comparisons of relative impact and strength of factors were conducted by economists who understood the economy well and who assumed the role of representative "experts". Obviously, the outcomes are heavily dependent on the quality of those judgments. As noted, the timing of the turnaround was conducted during the third week of December, 1991 and refined during first week of January, 1992. The estimation of the strength of the recovery was conducted during the second week of May, 1992.

Table 22 Matrix for subfactor importance relative to Conventional Adjustment influencing the Timing of Recovery

Which subfactor has the greater potential to influence Conventional Adjustment and how strongly?

| | | C | E | I | K | F | M | Vector Weights |
|---|---|---|---|---|---|---|---|---|
| Consumption | (C) | 1 | 7 | 5 | 1/5 | 1/2 | 1/5 | 0.118 |
| Exports | (E) | 1/7 | 1 | 1/5 | 1/5 | 1/5 | 1/7 | 0.029 |
| Investment | (I) | 1/5 | 5 | 1 | 1/5 | 1/3 | 1/5 | 0.058 |
| Confidence | (K) | 5 | 5 | 5 | 1 | 5 | 1 | 0.334 |
| Fiscal Policy | (F) | 2 | 5 | 3 | 1/5 | 1 | 1/5 | 0.118 |
| Monetary Policy | (M) | 5 | 7 | 5 | 1 | 5 | 1 | 0.343 |

For example, in Table 22, when comparing consumption with investment as a means of conventional adjustment, consumption is thought to be strongly more important and a 5 is entered in the first row and third column (1,3). Its reciprocal value of 1/5 is entered in the (3,1) position. On the other hand, when compared with confidence, consumption is not more important but confidence is strongly more important and a 1/5 is entered in the (1,4) position and a 5 in the (4,1) position. All other judgments follow this procedure. As before, the vector of weights is derived from the matrix as the principal eigenvector of the matrix.

Table 23 Matrix for subfactor importance relative to Economic Restructuring influencing the Timing of Recovery

Which subfactor has the greater potential to influence Economic Restructuring and how strongly?

| | | FS | DP | GC | Vector Weights |
|---|---|---|---|---|---|
| Financial Sector | (FS) | 1 | 3 | 3 | 0.584 |
| Defense Posture | (DS) | 1/3 | 1 | 3 | 0.281 |
| Global Competition | (GC) | 1/3 | 1/3 | 1 | 0.135 |

There are nine matrices that deal with the comparison of the time periods with respect to the nine factors above them (six for Conventional Adjustment and three for Economic Restructuring). The question to answer in making the judgments is: which time period is more likely to indicate a turnaround if the relevant factor is the sole driving force? We illustrate only with the first of these matrices.

Table 24 Relative importance of targeted time periods for **consumption** to drive a turnaround

| | 3 | 6 | 12 | 24 | Vector Weights |
|---|---|---|---|---|---|
| 3 months | 1 | 1/5 | 1/7 | 1/7 | .043 |
| 6 months | 5 | 1 | 1/5 | 1/5 | .113 |
| 12 months | 7 | 5 | 1 | 1/3 | .310 |
| 24 months | 7 | 5 | 3 | 1 | .534 |

Next we compare the two factors at the top: Conventional Adjustment (CA) and Economic Restructuring (R) in terms of the four time periods: Which is the most likely factor to dominate during a specified time period. Table 25 represents all four matrices for the four time periods.

Table 25 Which factor is more likely to produce a turnaround during the specified time period?

3 Months

| | CA | R | Vec. Wts. |
|---|---|---|---|
| CA | 1 | 5 | .833 |
| R | 1/5 | 1 | .167 |

6 Months

| | CA | R | Vec. Wts. |
|---|---|---|---|
| CA | 1 | 5 | .833 |
| R | 1/5 | 1 | .167 |

1 Year

| | CA | R | Vec. Wts. |
|---|---|---|---|
| CA | 1 | 1 | .500 |
| R | 1 | 1 | .500 |

2 Years

| | CA | R | Vec. Wts. |
|---|---|---|---|
| CA | 1 | 1/5 | .167 |
| R | 5 | 1 | .833 |

Now we group all the derived vector weights as columns in the appropriate positions of a matrix of mutual influences known as the supermatrix. For example, the first vector we derived from the matrix of subfactors of conventional adjustment is placed in the first column next to the six subfactors and under conventional adjustment. The factors are listed systematically so that the right vectors are listed to indicate the impact of the relevant factors on the left on the factors at the top. The supermatrix, being stochastic (with columns adding to one) is then raised to limiting powers to capture all the interactions and obtain the steady state outcome in which all columns within each block of factors are the same. We are particularly interested in the two identical columns at the bottom left corner of the matrix of Table 27. Either one is given by (0.224, 0.141, 0.201, 0.424).

**Table 26** The Supermatrix

| | C.A. | E.R. | Con. | Exp. | Inv. | Con. | F.P. | M.P. | F.S. | D.P. | G.C. | 3 mo. | 6 mo. | 1 yr. | ∃ 2 years |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Conven. Adjust | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.833 | 0.833 | 0.500 | 0.167 |
| Econ. Restruct. | 0.0 | 0.0 | 0.0 | 0.0. | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.167 | 0.167 | 0.500 | 0.833 |
| Consum. | 0.118 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Exports | 0.029 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Invest. | 0.058 | 0.0 | 0.0 | 0.00 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Confid. | 0.334 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Fiscal Policy | 0.118 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Monetary Policy | 0.343 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Financ. Sector | 0.0 | 0.584 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Defense Posture | 0.0 | 0.281 | 0.0 | 0.00 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Global Compet. | 0.0 | 0.135 | 0.0 | 0.00 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 3 months | 0.0 | 0.0 | 0.043 | 0.083 | 0.078 | 0.517 | 0.099 | 0.605 | 0.049 | 0.049 | 0.089 | 0.0 | 0.0 | 0.0 | 0.0 |
| 6 months | 0.0 | 0.0 | 0.113 | 0.083 | 0.078 | 0.305 | 0.086 | 0.262 | 0.085 | 0.085 | 0.089 | 0.0 | 0.0 | 0.0 | 0.0 |
| 1 year | 0.0 | 0.0 | 0.310 | 0.417 | 0.305 | 0.124 | 0.383 | 0.042 | 0.236 | 0.236 | 0.209 | 0.0 | 0.0 | 0.0 | 0.0 |
| ∃ 2 years | 0.0 | 0.0 | 0.534 | 0.417 | 0.539 | 0.054 | 0.432 | 0.091 | 0.630 | 0.630 | 0.613 | 0.0 | 0.0 | 0.0 | 0.0 |

**Table 27** The Limiting Supermatrix

| | C.A. | E.R. | Con. | Exp. | Inv. | Con. | F.P. | M.P. | F.S. | D.P. | G.C. | 3 mo. | 6 mo. | 1 yr. | ∃ 2 years |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Conven. Adjust | 0.0 | 0.0 | 0.484 | 0.484 | 0.484 | 0.484 | 0.484 | 0.484 | 0.484 | 0.484 | 0.484 | 0.0 | 0.0 | 0.0 | 0.0 |
| Econ. Restruct. | 0.0 | 0.0 | 0.516 | 0.516 | 0.516 | 0.516 | 0.516 | 0.516 | 0.516 | 0.516 | 0.516 | 0.0 | 0.0 | 0.0 | 0.0 |
| Consum. | 0.0 | 0.0 | 0.0 | 0.00 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.057 | 0.057 | 0.057 | 0.057 |
| Exports | 0.0 | 0.0 | 0.0 | 0.00 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.014 | 0.014 | 0.014 | 0.014 |
| Invest. | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.028 | 0.028 | 0.028 | 0.028 |
| Confid. | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.162 | 0.162 | 0.162 | 0.162 |
| Fiscal Policy | 0.0 | 0.0 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.057 | 0.057 | 0.057 | 0.057 | |
| Monetary Policy | 0.0 | 0.0 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.166 | 0.166 | 0.166 | 0.166 | |
| Financ. Sector | 0.0 | 0.0 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.301 | 0.301 | 0.301 | 0.301 | |
| Defense Posture | 0.0 | 0.0 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.145 | 0.145 | 0.145 | 0.145 | |
| Global Compet. | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.070 | 0.070 | 0.070 | 0.070 |
| 3 months | 0.224 | 0.224 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| 6 months | 0.151 | 0.151 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |
| 1 year 0.201 | 0.201 | 0.0 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | | |
| ∃ 2 years | 0.424 | 0.424 | 0.0 | 0.000.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | |

To obtain the forecast we multiply each value by the midpoint of its corresponding time interval and add (as one does when evaluating expected values). We have

$$.224 \times 1.5 + .151 \times 4.5 + .201 \times 9 + .424 \times 18 = 10.45 \text{ months}$$

from early Jan. 1, 1992. Note that at times the resulting supermatrix may not be stochastic which would then require weighting each cluster of factors as it impacts another cluster at the top.

The analyses for 2001 and 2008-2009 (two independent applications were made in this case and produced nearly identical answers; July 2010). We only give the figures for these forecasts. The first predicted turn around in eight and a half months, from April of that year which was precisely the date forecast by late in 2001.

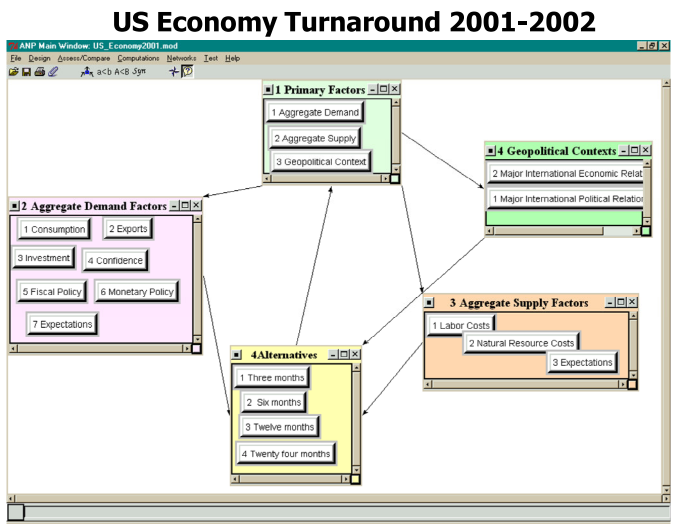



Figure 8 Factors and time periods that influenced the date of turnaround of the US economy in 2001

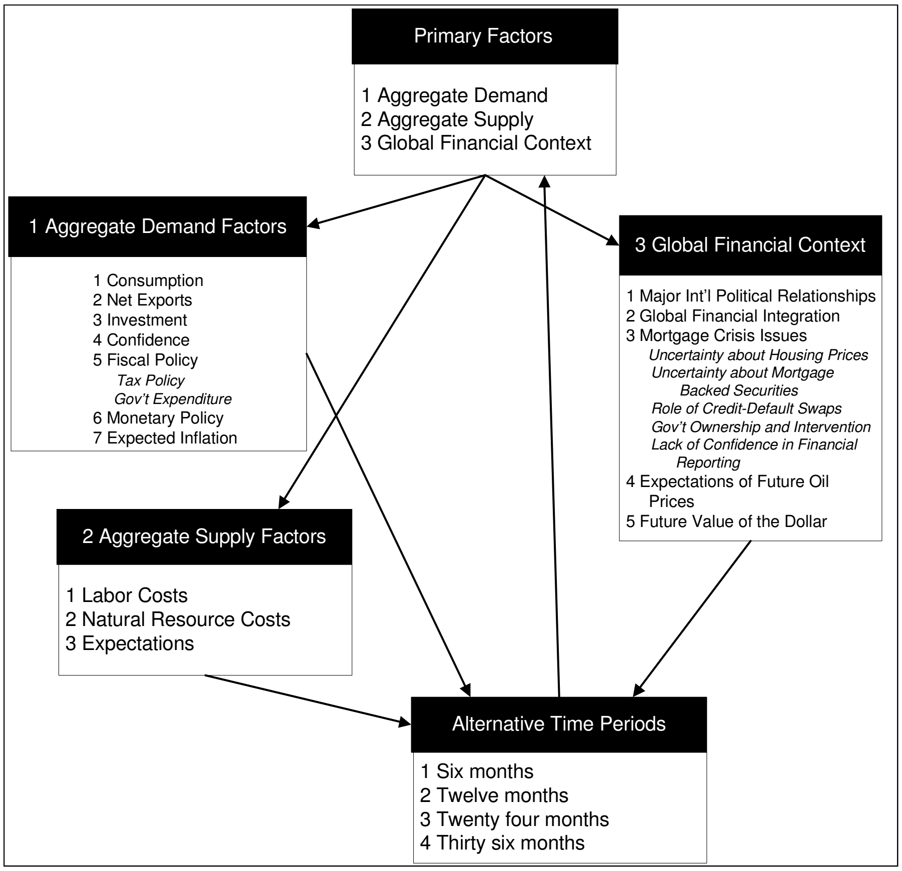



Figure 9 Factors and time periods that influenced the date of turnaround of the US economy in late 2008

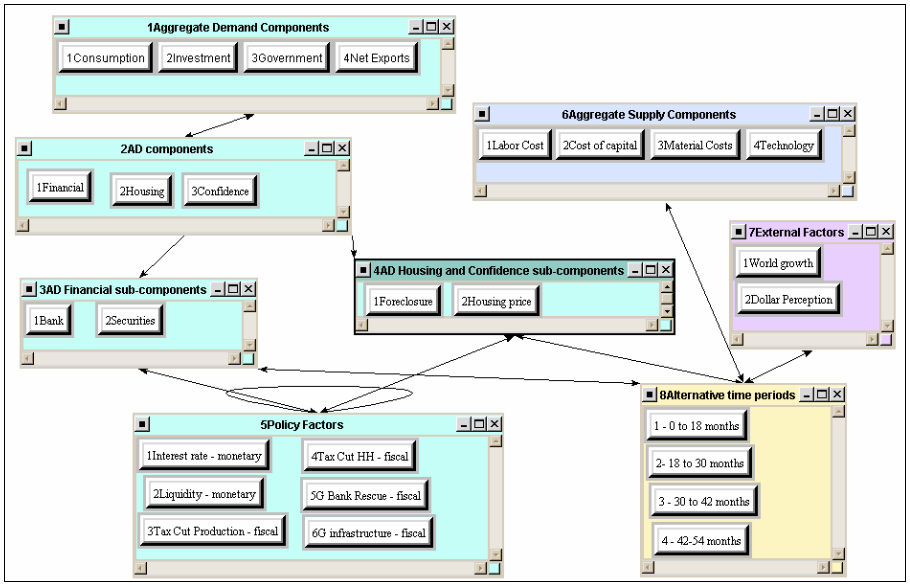



Figure 10 Factors and time periods that influenced the date of turnaround of the US economy in mid 2009

## 7. Conclusions

Because the importance of most intangible factors changes from situation to another and we need to know their priorities in these different situations in order to act accordingly, measurement and its interpretation will always depend on our judgments. Intangible criteria have no scales of measurement. To derive meaningful measurement for them, they must be compared in pairs. There is no other valid way to do it. Thus expert judgment is needed to make these comparisons and derive priorities of relative importance. This seems inevitable.

The AHP provides the opportunity to develop priorities for both tangible and intangible factors. As a result, it helps us explain the influences that shape outcomes by including all observable and known factors. The reader should try using it to make a simple personal decision to realize its power and usefulness.

There is an international society around the AHP, the International Symposium on the AHP (www.ISAHP.org) that meets biennially. The meeting places have ranged from China to Japan and Indonesia to the US to Chile, Switzerland, and Canada.

### Examples of Application since 2005

- Petrobras, Brazil's largest energy company, uses the AHP to help process and guide R&D investments. Petrobras has been recognized for its R&D prowess, pioneering new deep-water exploration and drilling capabilities that led Petrobras to the discovery and operationalization of a massive oil field on the Atlantic seabed.
- National Grid, an international energy delivery company with over 3 million electrical power customers in the U.S., uses AHP to guide its selection of new power transmission and delivery systems. These mission-critical systems are at the center of National Grid's operations, and represent a major selection that has a broad-ranging impact. Bill Tsolias, Manager of National Grid's Energy Management Group, said "We needed a decision process that would receive buy-in from all stakeholders... transmission, distribution, and IS." Decision Lens proven vendor selection process was used to successfully guide multiple groups within National Grid to the "best value" vendor.
- The Commander Navy Installations Command (CNIC) has selected Decision Lens as its decision making and budget allocation software application for enterprise alignment of shore installation support for all Navy Installations globally. CNIC is using AHP to prioritize all military construction projects and base of service activities to align investments and budget requests to strategic priorities and capabilities in support of all Navy operations. This is a $7B decision and resource allocation effort.
- The National Institutes of Health National Cancer Institute (NCI) used AHP to identify specific cancer vaccine target antigens for accelerated research. NCI developed a list of "ideal" cancer antigen criteria/characteristics and evaluated numerous representative antigens against those criteria for potential accelerated funding. The AHP software enabled NCI to capture input from academia, industry and government in an un-biased and structured way. NCI has recognized potential in some developmental cancer vaccines, and is interested in new approaches for identification, prioritization, and funding of translational cancer research.

### Examples of Application before 2002

Many people around the world are using AHP, not only Americans. Some of the applications mentioned here were done by non-Americans.

- In 1986 the Institute of Strategic Studies in Pretoria, a government-backed organization, used the AHP to analyze the conflict in South

Africa and recommended actions ranging from the release of Nelson Mandela to the removal of apartheid and the granting of full citizenship and equal rights to the black majority. All of these recommended actions were then quickly implemented.

- An oil company used it in 1987 to choose the best type of platform to build to drill for oil in the North Atlantic. A platform costs around 3 billion dollars to build, but the demolition cost was an even more significant factor in the decision.
- The process was applied to the U.S. versus China conflict in the intellectual property rights battle of 1995 over Chinese individuals copying music, video, and software tapes and CD's. An AHP analysis involving three hierarchies for benefits, costs, and risks showed that it was much better for the U.S. not to sanction China. The results of the study predicted what happened. Shortly after the study was complete, the U.S. awarded China most-favored nation trading status and did not sanction it.
- British Airways used it in 1998 to choose the entertainment system vendor for its entire fleet of airplanes.
- In 1999, the Ford Motor Company used the AHP to establish priorities for criteria that improve customer satisfaction. Ford gave Expert Choice Inc, an Award for Excellence for helping them achieve greater success with its clients.
- In 2001 it was used to determine the best site to relocate the earthquake devastated Turkish city Adapazari.
- IBM used the process in 1991 in designing its successful mid-range AS 400 computer. IBM won the prestigious Malcolm Baldrige award for Excellence for that effort. The book *The Silverlake Project* about the AS 400 project has a chapter devoted to how AHP was used in benchmarking in this effort.

A simple AHP hierarchy with final priorities is shown in Figure 1. The decision goal is to select the most suitable leader from a field of three candidates. Factors to be considered are age, experience, education, and charisma. According to the judgments of the decision makers, Janet is the most suitable candidate, followed by Jon and Alan. In this decision the three individuals are close and we need a fine structure to distinguish among their qualifications and make the necessary tradeoffs.

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